WS LHG	**Parabola** **From expanded form to vertex form**	Y10

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■ From $y = ax^2 + bx + c$ (***expanded form***) to $y = a(x-h)^2 + k$ (***vertex form***) and vice-versa:

- If you want to rewrite $y = a(x-h)^2 + k$ in the form $y = ax^2 + bx + c$, just expand!
- If you want to rewrite $y = ax^2 + bx + c$ in the form $y = a(x-h)^2 + k$, factorise a and then complete the square (*halve the coefficient of x, square it, add and subtract the result to* $y = ax^2 + bx + c$ *so a perfect square appears*). The vertex form makes it easy to find the vertex.

Particular case of FOIL: **Perfect Squares**	
○ Square of a sum: $(A+B)^2 = A^2 + 2AB + B^2$	E.g. : $(x+3)^2 = x^2 + 6x + 9$ (Use the *Square of a sum* formula with $A = x$ and $B = 3$)
○ Square of a difference: $(A-B)^2 = A^2 - 2AB + B^2$	E.g. : $(3x-7)^2 = 9x^2 - 42x + 49$ (use *Square of a difference* formula with $A = 3x$ and $B = 7$)

◯ **Exercise 1**. Use the perfect square formulas to rewrite the following parabolas in expanded form:

	Expanded form	Vertex form	The vertex is ...
(i)	Eg. $y = (x+2)^2 - 3 = x^2 + 4x + 4 - 3$. *Expand using the perfect square formula.* The expanded form is $y = x^2 + 4x + 1$.	$y = (x+2)^2 - 3$	$V(-2, -3)$ *Can be read from the expanded form. Can be checked using* $x_V = -\frac{b}{2a}$.
(ii)		$y = (x-3)^2 + 5$	
(iii)		$y = (x+7)^2 - 30$	
(iv)		$y = 2(x+5)^2 - 20$	

Answers are on OneNote

◯ **Exercise 2**. Complete the square to rewrite the following parabolas in vertex form:

	Expanded form	Vertex form	The vertex is ...
(i)	Eg. $y = x^2 + 6x + 5.$	$\frac{6}{2} = 3$ *and* $3^2 = 9$ *so* $x^2 + 6x + 5 = x^2 + 2 \times 3 \times x + 3^2 - 3^2 + 5$ $= (x + 3)^2 - 9 + 5 = (x + 3)^2 - 4$ [complete the square : *halve the coefficient of x (get 3), square it, add and subtract the result to* $y = ax^2 + bx + c$ *so a perfect square appears*] → The vertex form is $y = (x + 3)^2 - 4.$	$V(-3, -4)$ *Can be read from the expanded form. Can be checked using* $x_V = -\frac{b}{2a}.$
(ii)	$y = x^2 + 2x - 5$		
(iii)	$y = x^2 + 6x + 13$		
(iv)	$y = x^2 - 8x + 10$		
(v)	$y = x^2 + 3x + 10$		

Answers are on OneNote

ANSWERS

Exercise 1. Use the perfect square formulas to rewrite the following parabolas in expanded form:

	Expanded form	Vertex form	The vertex is ...
(i)	Eg. $y = (x+2)^2 - 3 = x^2 + 4x + 4 - 3$. *Expand using the perfect square formula.* The expanded form is $y = x^2 + 4x + 1$.	$y = (x+2)^2 - 3$	$V(-2, -3)$ *Can be read from the expanded form. Can be checked using* $x_V = -\frac{b}{2a}$.
(ii)	$y = x^2 - 6x + 14$	$y = (x-3)^2 + 5$	$V(3, 5)$
(iii)	$y = x^2 + 14x + 19$	$y = (x+7)^2 - 30$	$V(-7, -30)$
(iv)	$y = 2(x^2 + 10x + 25) - 20$ so $y = 2x^2 + 20x + 30$	$y = 2(x+5)^2 - 20$	$V(-5, -20)$

Exercise 2. Complete the square to rewrite the following parabolas in vertex form:

	Expanded form	Vertex form	The vertex is ...
(i)	Eg. $y = x^2 + 6x + 5$.	$\frac{6}{2} = 3$ *and* $3^2 = 9$ *so* $x^2 + 6x + 5 = x^2 + 2 \times 3 \times x + 3^2 - 3^2 + 5$ $= (x+3)^2 - 9 + 5 = (x-3)^2 - 4$ [complete the square : *halve the coefficient of x (get 3), square it, add and subtract the result to* $y = ax^2 + bx + c$ *so a perfect square appears*] → The vertex form is $y = (x+3)^2 - 4$.	$V(-3, -4)$ *Can be read from the expanded form. Can be checked using* $x_V = -\frac{b}{2a}$.
(ii)	$y = x^2 + 2x - 5$	$\frac{2}{2} = 1$ *and* $1^2 = 1$ *so* $y = x^2 + 2x + 1 - 1 - 5$ $y = (x+1)^2 - 6$	$V(-1, -6)$
(iii)	$y = x^2 + 6x + 13$	$\frac{6}{2} = 3$ *and* $3^2 = 9$ *so* $y = x^2 + 6x + 9 - 9 + 13$ $y = (x+3)^2 + 4$	$V(-3, 4)$
(iv)	$y = x^2 - 8x + 10$	$-\frac{8}{2} = -4$ *and* $(-4)^2 = 16$ *so* $y = x^2 - 8x + 16 - 16 + 10$ $= (x-4)^2 - 6$	$V(4, -6)$
(v)	$y = x^2 + 3x + 10$	*half the coeff of* $x = \frac{3}{2}$ *and* $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ *so* $y = x^2 + 3x + \frac{9}{4} - \frac{9}{4} + 10$ $y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 10 = \left(x + \frac{3}{2}\right)^2 + \frac{31}{4}$	$V\left(-\frac{3}{2}, +\frac{31}{4}\right)$

Answers are on OneNote