WorkSheet A	ALGEBRA	Y7 to Y12

✎ ✠ ✐ *Dr Laure Helme-Guizon* ✐ ✠ ✎ `http://bit.ly/ALG-WS-LHG` ✎ ✠ ✐

Table des matières

1 Algebraic Expressions

BIDMAS is watching you!
- First, **B** for **B**rackets
- Then, **I** for **I**ndices and roots
- Then, **MD** for **M**ultiplication and **D**ivision
- Finally, **AS** for **A**ddition and **S**ubtraction

Now that you know which operation you want to perform first, how do you do it correctly?

Forbidden or allowed?
- You can only add (or subtract) like terms. Remember that $x + x = 1x + 1x = 2x$ (not x^2).
- On the other hand, you can only multiply (or divide) any terms, even if they are not like terms. Remember that $x \times x = x^2$.

☐ **Exercise 1**
Simplify fully the following expressions :

a) $3 + 5x + 2 + 9x$
b) $3 \times 5x + 2 + 9x$
c) $3 + 5x \times 2 + 9x$
d) $3 \times 5x \times 2 + 9x$
e) $3 \times 5x \times 2 \times 9x$

☐ **Exercise 2**
Simplify fully the following expressions :

a) $-5x + 2 - 4x - 5$
b) $-5x \times 2 - 4x - 5$
c) $-5x - 4x \times -5$
d) $-5x \times 2 \times -4x$
e) $-5x + 2 \times -4x \times -5$

☐ **Exercise 3**
Simplify fully the following expressions :

a) $-3x + 7y + 8x - 5y$
b) $-3x \times 7y + 8x - 5y$
c) $-3x \times 7y + 8x \times -5y$
d) $-3x \times 7y \times 8x \times -5y$

☐ **Exercise 4**
Simplify $5z^8+5+3xy+6z^8+3+yx$

☐ **Exercise 5**
Simplify $2x\times y^2-8x^2\times 3y+5y\times -2xy$

☐ **Exercise 6** Simplify $\frac{-7x+4x}{-8x}$

1.1 Commutative, Associative properties

☐ **Exercise 7** Using the commutative property of multiplication, write an expression that is different but equivalent to $7+ab$.

☐ **Exercise 8** Using the commutative property of addition, write an expression that is different but equivalent to $3x+9$.

2 Substitution

☐ **Exercise 9**
Substitute $x=-2$ in $3x^2-(2x+3)$, and evaluate the result.

☐ **Exercise 10** Charlotte says that if you substitute $x=-3$ or $x=-2$ in $-x^2-(5x+4)$, you get the same result.
Is it true?

3 Expanding

3.1 Distributive Law [Y8]

☐ **Exercise 11**
Expand the following and simplify : $-7\,(7a-4)+3$

☐ **Exercise 12**
Simplify fully the following expressions :

a) $2x+10+6\,(x+4)$

b) $3x+8+2\,(6x+1)$

☐ **Exercise 13** Expand both brackets and then fully simplify the following expressions :

a) $x\,(x+7)+2\,(x+10)$

b) $x\,(x+2)+5\,(7x+2)$

☐ **Exercise 14**
Expand and simplify each of the following :

a) $7\,(n+1)-4\,(n+3)$

b) $7\,(w-6)-3\,(w-1)$

Note that you can interpret $7\,(n+1)-4\,(n+3)$ as meaning $7\,(n+1)+(-4)\times(n+3)$ or as meaning $7\,(n+1)-(+4)\times(n+3)$. In both cases you will get the same result. I recommend the first interpretation as you get the result with one step less.

A negative sign in front of a bracket or a fraction bar ...
... can be removed provided you change *all* the signs inside the bracket or the numerator of the fraction.
E.g. : $2x^3+9-(3-5x+x^2)=2x^3+9-3+5x-x^2$.
E.g. : $9x-7-\frac{-6x^2-7x+4}{3}=9x-7+\frac{6x^2+7x-4}{3}$.
If it helps, you may think that you are distributing a -1 using the Distributive Law.

☐ **Exercise 15**
Simplify $3s+5-(2s+8)$.

☐ **Exercise 16**
Expand the following and simplify : $3(2x-5)-(7x+4)$

☐ **Exercise 17**

a) Expand and simplify $-2(2x-5)-(2-5x)$

b) Without a calculator (ideally, doing mental maths), evaluate $-2(2x-5)-(2-5x)$ when $x=-12$.

☐ **Exercise 18**
Simplify $2x(5-x)-\left(5x^2+3x-7\right)$.

☐ **Exercise 19**
Simplify $x-2y+3t-\left(4x+5y-6t\right)$.

☐ **Exercise 20**
Simplify $-\left(4+2\left(x+8y\right)\right)+6\left(-2x+y\right)$.

☐ **Exercise 21**
Simplify $-\left(6-2\left(x-7y\right)\right)+6\left(-3x+y\right)$.

☐ **Exercise 22** Expand the brackets and simplify the following : $3a\,(a-6)-\left(a^2-3a+7\right)$

☐ **Exercise 23**
Let $A=(3x+2)\,(3x-5)-(-7-9x)$

a) Expand the brackets and simplify A.

b) Without a calculator, prove that A is an integer when $x=\frac{1}{3}$.

☐ **Exercise 24**
Expand and simplify $2a\left(2a^2-5a+10\right)-a^2\,(a+1)$.

☐ **Exercise 25**
Expand and simplify $2t\,(t-3)+3-t\,(t+6)$.

☐ **Exercise 26**
Expand and simplify $4y\left(2y^2-7y+6\right)-y^2\left(y+9\right)$.

3.2 Product of Binomials : FOIL

☐ **Exercise 27** Expand the brackets and simplify the following [Use "FOIL"] : $(x+2)(x+10)$

☐ **Exercise 28** Expand the brackets and simplify the following [Use "FOIL"] : $(3x+2)(x-10)$

☐ **Exercise 29**
Expand and simplify $(2m+6)(m-6)$.

☐ **Exercise 30**
Expand and simplify $(7y-6)(6y+6)$.

☐ **Exercise 31**
Expand and simplify $(5y-3)(-4y-1)$.

☐ **Exercise 32**
Expand and simplify $5(-3x-5)(5x+1)$.

☐ **Exercise 33**
Expand and simplify $2(y+8)(y+7)$.

☐ **Exercise 34**
Expand and simplify $-8(-6x-9)(x+2)-168x$.

☐ **Exercise 35**

a) Expand $3x^2y(7y-9x)$.
b) Expand $3x^2+y(7y-9x)$
c) Expand $(3x^2+y)(7y-9x)$

☐ **Exercise 36**

a) Expand and simplify $(x+3)(2x-5)-10x$.
b) Guess (mental maths only!) what you get when you expand and simplify $10x-(x+3)(2x-5)$
c) Check your guess by expanding and simplifying $10x-(x+3)(2x-5)$

☐ **Exercise 37**
Expand and simplify $2m^2-(2m+4)(m-2)$ to prove that this expression doesn't depend on$m$.

3.3 Special Expansions : Perfect squares and difference of squares

☐ **Exercise 38**
Expand and simplify the following expressions

a) $(x+3)^2$.
b) $(3x-5y)^2$.
c) $(3x-5y)(3x+5y)$.
d) $(x+\frac{1}{x})^2$.

☐ **Exercise 39**
Expand and simplify the following expressions

a) $(4y-5)^2$.
b) $(x+3)(x-3)$.
c) $(3m-7)(2m-4)-(m^2+15m+8)$.

☐ **Exercise 40**
Expand and simplify the following expressions

a) $(t+3)(t-3)$
b) $(10x-7y)(10x+7y)$
c) $\left(y+\frac{1}{4}\right)\left(y-\frac{1}{4}\right)$.
d) $\left(7x-\frac{5}{3}\right)\left(7x+\frac{5}{3}\right)$.

☐ **Exercise 41**
Let $T=(3x+2)^2-(9x-2)(x+4)$.

a) Expand and simplify T.
b) Evaluate (without a calculator!) T when $x=\frac{4}{11}$.

☐ **Exercise 42** Expand the brackets and simplify the following : $(x+12)(x-2)$

☐ **Exercise 43** Expand $(x+3)^2$

☐ **Exercise 44** Expand $(x-3)^2$

☐ **Exercise 45** Expand $(x-9)^2$

☐ **Exercise 46** Expand $(3x+5)^2$

☐ **Exercise 47** Expand $(3x-4y)^2$

☐ **Exercise 48** Expand $(7x+t)^2$

☐ **Exercise 49** Expand $(3x-4x)^2$

☐ **Exercise 50**
Expand and simplify $R=(x-3)^2-(x-1)(x-2)$ and derive from this result (without a calculator) the exact value of $99997^2-99999\times 99998$.

4 Writing Expressions

☐ **Exercise 51**
A square of side lengths measuring $x+3$ centimetres has each side enlarged by a factor of 3. Write an expression, in expanded form, for the area of the new square.

☐ **Exercise 52**
A square with a side length of $u-4$ centimetres has each side enlarged by a factor of 5. Write an expression for the area of the new square in expanded form.

☐ **Exercise 53**
Dave is trying to determine the dimensions of a rectangular box that will result in the largest volume. If the height of the box is 2 centimetres, the width is $7y$ centimetres and the length is 2 centimetres more than double the width, find an expression in expanded form for the volume of the box in terms of y.

5 Factorisation

☐ **Exercise 54** Factorise $6t^3s - 64s^2t^2$.

☐ **Exercise 55** Factorise $\frac{3x}{7} + \frac{2y}{7}$.

☐ **Exercise 56** [Y8]
Factorise fully (=as much as possible) each of the following :

a) $x^2 - 7x$
b) $32at^5 - 48a^7t^3$

☐ **Exercise 57** Factorise each of the following :

a) $x^2 - 9$
b) $x^2 + 6x + 8$
c) $6x^2 + 13x - 5$

☐ **Exercise 58** [Y10/11]
Factorise fully each of the following :

a) $7x^2 - 63$
b) $11x^2 - 11x - 22$
c) $x^4 - 16$

☐ **Exercise 59**
Factorise the expression $S = (7x-3)^2 - 9$ and derive from this result (without a calculator!) the exact value of S when $x = \frac{1}{7}$.

6 Algebraic Fractions

6.1 Let's start with numerical fractions

☐ **Exercise 60**
Simplify the following fractions without a calculator :

a) $\frac{35}{45}$
b) $\frac{12}{18}$
c) $\frac{88}{99}$
Challenge!
d) $\frac{3\times5^3}{7\times5^2}$
e) $\frac{3\times7^4}{7^2\times12}$
f) $\frac{3\times x^3}{7\times x^2}$
g) $\frac{3y^4}{12y^2}$

6.2 Simplifying Algebraic Fractions

How to simplifying algebraic fractions?

- First, factorise the numerator and the denominator
- Then, cancel common factors, i.e. divide the top and the bottom by the common factors.

You may *not* cross out if you have '+' or a '-'. What you need is a '×'. For instance, you may not cross the 4's in $\frac{x^2+4}{3x+4}$ but you could cross them out in $\frac{x^2\times4}{3x\times4}$.

☐ **Exercise 61**
Simplify the following fractions without a calculator :

a) $\frac{8m}{72}$
b) $\frac{2x}{14}$
c) $5z^6 \div 2z^2$ (Hint : Convert to fraction form and *then* simplify)
d) $\frac{-6m^2p}{-8mp}$
e) $\frac{2u\times8vw}{8v\times6u}$

Simplifying Fractions : 1 or −1 ?

a) $\frac{a}{a} = 1$ for any number $a \neq 0$.
b) $\frac{a}{-a} = \frac{-a}{a} = -\frac{a}{a} = -1$ for any number $a \neq 0$.

Example : $\frac{x-y}{y-x} = \frac{x-y}{-(x-y)} = -1$

☐ **Exercise 62** Simplify $\frac{4t-8}{2}$

☐ **Exercise 63** Simplify $\frac{30t^7}{15t^3-40t}$

☐ **Exercise 64** Simplify $\frac{3x-21}{6}$

☐ **Exercise 65** Simplify $\frac{12}{3x-21}$

☐ **Exercise 66** Simplify $\frac{3x-21}{2x-14}$

☐ **Exercise 67** Simplify $\frac{y+8}{y^2-64}$

☐ **Exercise 68** Simplify $\frac{a^2-1}{a+1}$

☐ **Exercise 69** Simplify $\frac{a^2-49}{a-7}$

☐ **Exercise 70** Simplify $\frac{2x^2-32}{x-4}$

☐ **Exercise 71** Simplify $\frac{x-5}{2x^2-7x-15}$

☐ **Exercise 72** Simplify $\frac{x^2-25}{2x^2-7x-15}$

☐ **Exercise 73** Simplify $\frac{3(7x-6)-8x+5}{26}$

☐ **Exercise 74** Simplify $\frac{3(7x-6)-(17x+10)}{2}$

☐ **Exercise 75** Simplify $\frac{x^2-5x+7}{-5x+7+x^2} - 3$

☐ **Exercise 76** Simplify $\frac{x^2+5x-3}{-6+2x^2+10x}$

☐ **Exercise 77** Simplify $\frac{x^2+5x-3}{-3+x^2+5x}$

☐ **Exercise 78** Simplify $\frac{x-3}{3-x}$

6.3 Multiplying and dividing Algebraic Fractions

☐ **Exercise 79** [*From Y8 program*]
Simplify $\dfrac{a^2b^4}{6} \times \dfrac{9}{a^2b^2}$

☐ **Exercise 80** [*Y8*]
Fill in the box to make the statement true :
$21p^3q^7 \times \boxed{} = 105p^7q^{20}$

☐ **Exercise 81** [*Y8*]
Simplify : $\frac{72x^5y}{21} \div \frac{x^2}{14y^2}$

☐ **Exercise 82** [*2015 Y8 CT1*]
Fill in the box to make the statement true :
$20p^3q^7 \div \boxed{} = 5p^2q^5$

☐ **Exercise 83** [*From Y8 program*]
Simplify $\dfrac{3x^2}{8y^5} \div \dfrac{15x^3}{4y}$

☐ **Exercise 84**
Simplify $\frac{12x^7}{8(xy)^5} \div \dfrac{15x^3}{45y}$

☐ **Exercise 85**
Simplify : $\frac{3x+3}{x^2-2x} \times \frac{x^2+x-6}{x^2-1}$

☐ **Exercise 86** [2014 Y9 Y]
Factorise and simplify :

a) $\frac{2a+8}{a+4}$

b) $\frac{x+2}{x-3} \times \dfrac{x^2-5x+6}{x^2-4}$

☐ **Exercise 87** [From Y9 programme]
Simplify $\frac{3m-6}{4} \times \frac{8m}{m^2-2m}$

☐ **Exercise 88**
Simplify : $\frac{3x+3}{x^2-2x} \times \frac{x^2+x-6}{x^2-1}$

☐ **Exercise 89**
Simplify : $\frac{x^2-2x-24}{x^2-1}$

☐ **Exercise 90**
Simplify : $\frac{3x+3}{x^2-2x} \div \frac{x^2-1}{x^2+x-6}$

6.4 Adding Algebraic Fractions, Same Denominators

How to add fractions?

- We can add and subtract fractions *only* when they have a common denominator.
- To get common denominators, we can multiply both top and bottom of a fraction by the same amount.

☐ **Exercise 91** Simplify $\frac{-3xy}{3xy} - \frac{5}{3}$

☐ **Exercise 92** Simplify $\frac{8x+9}{6} + \frac{7x-4}{6}$

☐ **Exercise 93**
Simplify $\frac{2p^2+7p+6}{p+8} - \frac{3p^2+4p+5}{p+8}$

☐ **Exercise 94** Simplify $\frac{3y^2+7y+5}{7} - \frac{6y^2+3}{7}$

☐ **Exercise 95** Simplify $\frac{8v+5}{v-4} - \frac{5v+5}{v-4}$

☐ **Exercise 96** Simplify $\frac{-9x-6\times5x}{-10x}$

☐ **Exercise 97**
Simplify $\frac{3p^2+4p+3}{p-9} - \frac{2p^2+6p+6}{p-9}$

☐ **Exercise 98** Simplify $\frac{3p^2+9}{p+5} - \frac{4p^2+5p}{p+5}$

☐ **Exercise 99**
Simplify $\frac{4n^2+7n+3}{n^2+n+8} - \frac{9n^2+5n+5}{n^2+n+8}$

6.5 Adding Algebraic Fractions, Unlike Denominators

☐ **Exercise 100**
What is the lowest common denominator of the fractions of the fractions $\frac{1}{5}$ and $\frac{1}{4}$?

☐ **Exercise 101**
What is the lowest common denominator of the fractions of the fractions $\frac{1}{12}$ and $\frac{1}{60}$?

☐ **Exercise 102**
What is the lowest common denominator of the fractions of the fractions $\frac{1}{12}$ and $\frac{1}{30}$?

☐ **Exercise 103**
What is the lowest common denominator of the fractions of the fractions $\frac{1}{12y}$ and $\frac{7x}{y^2}$?

☐ **Exercise 104** Lola, Amber and Sophie were discussing how to find the lowest common denominator for the two fractions $\frac{1}{(x-2)(x+8)}$ and $\frac{1}{(x+8)^2}$. Who has the correct explanation?

Ⓐ Lola said that the words 'lowest' and 'common' are the key. The lowest part that is common to both is $(x+8)$, so that will be the lowest common denominator.

Ⓑ Amber said that these two fractions have no lowest common denominator.

Ⓒ Sophie said that the result has to end up with identical denominators. The lowest product that contains all of the factors is $(x-2)(x+8)^2$, so that will be the lowest common denominator.

☐ **Exercise 105** For the following pairs of fractions, determine whether or not the given lowest common denominator is correct.

a) For the fractions $\frac{2}{x}$and $\frac{7}{x^2}$, the lowest common denominator would be x^2. TRUE or FALSE?

b) For the fractions $\frac{3}{x^2(x+2)}$ and $\frac{7}{(x+2)^2}$, the lowest common denominator would be $x(x+2)$. TRUE or FALSE?

c) For the fractions $\frac{4}{(x+6)^2(x-4)}$ and $\frac{7}{(x+6)^3}$, the lowest common denominator would be $(x+6)^3(x-4)$. TRUE or FALSE?

☐ **Exercise 106** For the following pairs of fractions, determine whether or not the given lowest common denominator is correct.

a) For the fractions $\frac{5}{x^3}$ and $\frac{1}{x^2}$, the lowest common denominator would be x^2. TRUE or FALSE?

b) For the fractions $\frac{3}{(x+5)^2(x-3)}$ and $\frac{2}{(x+5)^3}$, the lowest common denominator would be $(x+5)^3(x-3)$. TRUE or FALSE?

c) For the fractions $\frac{3}{x^2(x+2)}$ and $\frac{7}{(x+2)^2}$, the lowest common denominator would be $x^2(x+2)$. TRUE or FALSE?

☐ **Exercise 107** What is the lowest common denominator of the fractions of the fractions $\frac{1}{12y^4z^2}$ and $\frac{1}{18y^2z^3}$?

☐ **Exercise 108** What is the lowest common denominator of the fractions $\frac{1}{9x^2z^5}$ and $\frac{1}{21x^3z^3}$?

☐ **Exercise 109** Simplify $\frac{4}{9xz} + \frac{1}{36xy}$.

☐ **Exercise 110** Simplify $\frac{2x+1}{45} + \frac{3x+5}{30}$.

☐ **Exercise 111** Simplify $\frac{7p-q}{7} - \frac{p+6q}{5}$.

☐ **Exercise 112** Simplify $\frac{a+7}{a^2b} + \frac{b-5}{ab^2}$.

☐ **Exercise 113** [2014 Y9 Y]
Simplify:

a) $\frac{x}{2} - \frac{x}{5}$

b) $\frac{1}{x+1} + \frac{1}{x-1}$

☐ **Exercise 114** Simplify $\frac{-3x^2}{6x} - \frac{5x}{2}$

☐ **Exercise 115** Simplify $\frac{2}{5p} - \frac{4}{p}$.

☐ **Exercise 116** Simplify $\frac{5x}{9(x+1)} - \frac{5}{x+1}$.

☐ **Exercise 117** Simplify $\frac{4x}{5(x+4)} - \frac{4}{x+4}$.

☐ **Exercise 118** Simplify $\frac{6x}{11(x+5)} - \frac{3}{x+5}$.

☐ **Exercise 119** Simplify $\frac{1}{6} + \frac{6}{x-6}$.

☐ **Exercise 120** Simplify $\frac{a+2}{a^7b^5} - \frac{b^4+3}{a^6b^9}$.

☐ **Exercise 121** [Y8]
Express as a single fraction $\frac{a}{4} + \frac{2a+1}{3} - a$.

☐ **Exercise 122** [Y8 Yearly 2017]
Express as a single fraction $\frac{a}{4} - \frac{2a+1}{3} + a$.

☐ **Exercise 123** Simplify $\frac{3}{x+7} - \frac{1}{x-7}$.

☐ **Exercise 124** [From Y9 5.3 Programme]
Factorise the denominators and then simplify:
$\frac{4}{x^2+x} - \frac{3}{x^2-1}$

☐ **Exercise 125** [From Y9 5.3 Programme]
Simplify: $\frac{4}{x^2-9} + \frac{2}{3x+9}$

☐ **Exercise 126** Factorise the denominators and then simplify: $\frac{x-3}{x^2-16} - \frac{1}{x+4}$

☐ **Exercise 127** Simplify $\frac{9}{2y+6} + \frac{3}{4y-10}$.

☐ **Exercise 128** Factorise the denominators and then simplify: $\frac{x}{x^2-49} - \frac{7}{x+7}$.

☐ **Exercise 129** Simplify $\frac{2x-25}{x^2-81} - \frac{3}{x+9}$.

☐ **Exercise 130** Simplify $\frac{5}{x+5} - \frac{3}{(x-2)(x-9)}$.

☐ **Exercise 131**
Simplify: $\frac{5}{(x-1)(x-9)} + \frac{2}{(x-1)(x+7)}$

☐ **Exercise 132** Factorise the denominators and then simplify: $\frac{8}{3x+24} + \frac{3}{x^2+8x}$

☐ **Exercise 133** Factorise the denominators and then simplify: $\frac{x+9}{x^2+14x+45} + \frac{x-8}{(x+5)(x+8)}$

☐ **Exercise 134** Factorise the denominators and then simplify: $\frac{x+7}{x^2+18x+77} + \frac{x-10}{(x+11)(x+10)}$

☐ **Exercise 135** Write as a single simplified fraction: $\frac{x+7}{x^2+18x+77} + \frac{x-10}{(x+11)(x+10)}$

☐ **Exercise 136** Factorise the denominators and then simplify : $\frac{x^2+10x+21}{x^2-49} - \frac{x+5}{x^2-2x-35}$

☐ **Exercise 137** Write as a single simplified fraction : $\frac{x^2+10x+21}{x^2-49} - \frac{x+5}{x^2-2x-35}$

☐ **Exercise 138** Factorise the denominators and then simplify : $\frac{1}{x^2-4x-32} - \frac{3}{x^2-64}$

☐ **Exercise 139** Factorise the denominators and then simplify : $\frac{2}{x^2+7x-18} + \frac{3}{x^2-6x+8}$

☐ **Exercise 140** In the expression $\frac{1}{x}$, what value(s) are not allowable as substitutions for the variable x?

☐ **Exercise 141** In the expression $\frac{1}{x+3}$, what value(s) are not allowable as substitutions for the variable x?

6.6 Compound Fractions

☐ **Exercise 142** Simplify $\dfrac{1}{\frac{1}{a}+\frac{1}{b}}$

☐ **Exercise 143** Simplify $\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}}$

☐ **Exercise 144** Prove that $\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}} + \frac{b+a}{a-b}$ does not depend on the values chosen for a and b.

7 First Degree Equations

How can I tell if my equation is a degree one equation?

If you were to expand and simplify, would you get only x's and numbers (No $\frac{1}{x}$, no x^2, no x^3,...etc left after simplification)?

If the answer is YES, then it is a degree 1 equation.

How to solve a degree one equation?

a) Do some clean up :
   i) If all the terms are say, multiples of 10, divide both sides of the equation by 10.
   ii) If there are some denominators, get rid of them : multiply both sides by a common multiple of the denominators
b) Now solve :
   i) Put all the x's on one one side (you may need to expand sone brackets first)
   ii) Put all the *constants* (= numbers without x) on the other side.
   iii) Divide both sides by the coefficient of x.

How to deal with Equations?

a) You can add (or subtract) the *same* number to *both* sides of an equation.
b) You can multiply (or divide) *both* sides of an equation by the *same* number, provided this number is not zero.

Note that subtracting is really the same as adding the opposite (Eg.$5-8=5+(-8)$) and this is why the rules are the same for addition and subtraction.
Note that dividing is really the same as multiplying by the reciprocal (Eg. $7 \div 2 = \frac{7}{2} = 7 \times \frac{1}{2}$) and this is why the rules are the same for multiplication and division.

While Solving Equations

In every single line make sure you can articulate what, exactly, you are doing to both sides of the equation. If you can't articulate it, it cannot possibly be a legitimate thing to do!

☐ **Exercise 145**
Solve the equation $-3x+5=12+4x$.

☐ **Exercise 146** [Y7, Beginning of Equations]
Solve the following equations (that means that you need to find the value of the letter) :

a) $x+4=12$
b) $x+12=4$
c) $3x=741$
d) $\frac{x}{3}=6$
e) $9x=72$
f) $9x=-36$
   [Getting harder : I am just trying to convince you that (1) solutions do NOT have to be integers, fractions are just fine and (2) guessing won't always work, we need better tools.]
g) $10x=3$
h) $x+6=2.5$
i) $3x=-4$
j) $3x+1=-12.5$

☐ **Exercise 147**
Solve the equation $-3x+5=12$.

NB :
(1) Solutions do NOT have to be integers, fractions are just fine
(2) Guessing won't always work!
(3) Write your answer in formal setting out showing all the steps.

☐ **Exercise 148** [*From Y7 Program*]
Solve the following equations (that means that you need to find the value of the letter) :

a) $x-7=15$

b) $2x-7=15$

c) $7-2x=15$

d) $\frac{x}{7}=5$

e) $\frac{2x}{7}=5$

☐ **Exercise 149** [*From Y7 Program*]
Find several equations with solutions $x=5$.

☐ **Exercise 150** Consider the equation $2(x-7)=12x-9$.

a) Is $x=-4$ a solution of this equation?

b) Is $x=-\frac{1}{2}$ a solution of this equation?

☐ **Exercise 151** Consider the equation $2x^2+10x-20=3x-5$.

a) Is $x=-5$ a solution of this equation?

b) Is $x=3$ a solution of this equation?

c) Is $x=\frac{3}{2}$ a solution of this equation?

☐ **Exercise 152**
Solve the equation $\frac{5x}{2}-2=3$.

7.1 degree one, var on both sides, Distributive Law

☐ **Exercise 153**
Solve the equation $-3x+4-5x+8=12x$.

☐ **Exercise 154**
Solve the equation $-x+8=-11$.

☐ **Exercise 155** [2015 Y9 Y]
Solve the following equations :

a) $3x+4=10$

b) $9x-4=2(2x+8)$

c) $\frac{x}{8}-7=6$

☐ **Exercise 156**
Solve the equation $6x(x-2)-2x(3x-7)=14$.

☐ **Exercise 157**
Solve the equation $-3(2x+5)+7(3x+10)=10x$.

7.2 First, multiply or divide the equation

☐ **Exercise 158**
Solve the equation $100x+800=900x-700$.

☐ **Exercise 159**
Solve the equation $60x+90=30x-150$.

☐ **Exercise 160**
Solve the equation $4(x+5)=16x-40$.

☐ **Exercise 161**
Solve the equation $\frac{9x-15}{4}+2=2x$.

☐ **Exercise 162**
Solve the equation $\frac{4x-1}{5}+6x=2x+\frac{14}{5}$.

☐ **Exercise 163**
Solve the equation $\frac{2x+1}{3}=2x+\frac{1}{3}$.

☐ **Exercise 164**
Solve the equation $\frac{x+1}{3}+\frac{5x-2}{6}=x+\frac{7}{6}$.

☐ **Exercise 165**
Solve the equation $\frac{-70x-3}{7}+\frac{5x+2}{2}=8-x$.

☐ **Exercise 166**
Solve the equations

a) $16x-32=160+48x$

b) $100(2x+3)=500-400x$

c) $8(12-20x)=40x+32$

☐ **Exercise 167**
Solve the equation $\frac{x-13}{2}-\frac{7x-3}{2}=3x-\frac{7}{2}$.

☐ **Exercise 168**
Solve the equation $\frac{x+1}{3}-\frac{5x-2}{6}=x+\frac{7}{6}$.

☐ **Exercise 169** [Y9 and up]
Solve the equation $(3x-5)^2-(3x+5)^2=14$.

☐ **Exercise 170** [2014 Y9 Y]
Solve :

a) $x-4=3x+8$

b) $3(x+5)=x$

c) $\frac{x+8}{3}=\frac{x-2}{2}$

d) $9x^2=81$

A negative sign in front of a bracket or a fraction bar ...
... can be removed provided you change *all* the sign inside the bracket or the numerator of the fraction.
E.g. : $2x^3+9-(3-5x+x^2)=2x^3+9-3+5x-x^2$.
E.g. : $9x-7-\frac{-6x^2-7x+4}{3}=9x-7+\frac{6x^2+7x-4}{3}$.

☐ **Exercise 171**
Solve the equation $-3(2x-7)-(3x+10)=10-x$.

☐ **Exercise 172**
Solve the equation $\frac{x-7}{2}+\frac{5x-3}{2}=4x-\frac{11}{2}$.

☐ **Exercise 173**
Solve the equation $\frac{x-7}{2}+\frac{5x-3}{2}=3x-\frac{11}{2}$.

☐ **Exercise 174**

a) Solve the equation $\frac{3}{x-2}=\frac{4}{2x-5}$.
b) Solve the equation $\frac{3}{\sqrt{x}-2}=\frac{4}{2\sqrt{x}-5}$

7.3 Word problems leading to degree 1 equations

☐ **Exercise 175** The dimensions of a rectangle are $3y+2$ cm by $5y+7$ where y is an unknown number. It perimeter is $128+5y$ cm. Find y. Find the dimensions of the rectangle.

☐ **Exercise 176** A Payroll Officer has been told to distribute a bonus to the employees of a company worth 17% of the company's net income. Since the bonus is an expense to the company, it must be subtracted from the income to determine the net income. If the company has an income of \$180000 before the bonus, then the Payroll Officer must solve :
$B=0.17\,(180000-B)$ to find the bonus B.

☐ **Exercise 177**
When a number is added to both the numerator and denominator of $\frac{1}{4}$, the result is $\frac{2}{3}$. Let n represent the number. Solve for n.

Harder...

☐ **Exercise 178**
Solve the equation $\dfrac{1}{x}+\dfrac{1}{3}=\dfrac{1}{5}$.

8 Degree one Inequations

☐ **Exercise 179**
Solve the inequation $2(x-5)+7>5x-2$.

9 Quadratics

9.1 Easy Quadratics and Cubics Y8 and 9

☐ **Exercise 180** Solve $x^2=25$.

☐ **Exercise 181** Solve $x^2+2x=2x+1$.

☐ **Exercise 182** Solve $x^2=7$.

☐ **Exercise 183** [2015 Y9 Y]
Solve $2x^2=50$.

☐ **Exercise 184** Solve $2x^2=128$.

☐ **Exercise 185** Solve $\frac{x^2}{3}=27$.

☐ **Exercise 186**
Solve $6x(x+3)=2x(3x-9)-45$.

☐ **Exercise 187**

a) Solve $5x^2-320=0$
b) Solve $5x^2+320=0$
c) Solve $5x^3-320=0$
d) Solve $5x^3+320=0$

✐ ✠ **QUADRATICS and other equations** [Y10+] ✠ ✐

9.2 Zero product property

By "other equations", I mean that in order to solve equations, you may need to (1) put everything on one side, (2) factorise and (3) to use the following fact :

$A\times B=0$ if and only if $A=0$ or $B=0$.

☐ **Exercise 188**
[*Zero product property for beginners*]

a) Factorise $3x(7x-1)-6(7x-1)$
b) Solve $3x(7x-1)-6(7x-1)$.

☐ **Exercise 189**
[*Zero product property for beginners*]

a) Factorise $2x^2-13x-7$.
b) Solve $2x^2-13x-7=0$.

☐ **Exercise 190**

a) Factorise $x^2-25-4x(x-5)$
b) Solve $x^2-25-4x(x-5)=0$.

☐ **Exercise 191**

a) Factorise $x^2-9-4x(x+3)$
b) Hence solve $x^2-9-4x(x+3)=0$.

☐ **Exercise 192**
Solve $3x^3+5x^2-2x=0$ by factorising.
No other method may be used.

9.3 Completing the square

☐ **Exercise 193**
Solve

a) $(x+2)^2 = 121$

b) $(3x-7)^2 = 81$

☐ **Exercise 194**
Solve

a) $(x+3)^2 = 49$

b) $(2x+1)^2 = 25$

☐ **Exercise 195**
Find the numbers that "complete the square" in each equation

a) $x^2 + 6x + \ldots = (x+\ldots)^2$.

b) $x^2 - 8x + \ldots = (x-\ldots)^2$.

c) $x^2 - 5x + \ldots = (x \ldots \ldots)^2$.

☐ **Exercise 196**
Find the numbers that "complete the square" in each equation so that there are no x's outside of the bracket.

a) $x^2 + 10x = (x+\ldots)^2 - \ldots$.

b) $x^2 - 12x = (x \ldots \ldots)^2 - \ldots$.

☐ **Exercise 197**
Find the numbers that "complete the square" in each equation so that there are no x's outside of the bracket.

a) $x^2 + 16x = (x \ldots \ldots)^2 - \ldots$.

b) $x^2 - 9x = (x \ldots \ldots)^2 - \ldots$.

☐ **Exercise 198**
Solve $x^2 + 14x + 1 = 0$ by completing the square.

9.4 Quadratic formula

Quadratic Formula
The solutions of the quadratic equation $ax^2 + bx + c = 0$
are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

TIP! When solving a quadratic, try to guess the factorisation for about 30 seconds (using PSF) and if this fails, use the quadratic formula. Guessing the factorisation, when possible, is much faster.

☐ **Exercise 199**
Solve $(2x+1)^2 - (3x-7)^2 = 0$ in two different ways.

☐ **Exercise 200**

a) Factorise fully $-8x^2 - 80x$.

b) Expand and simplify $8(-x+2)(x+12) - 192$.

c) Solve $8(-x+2)(x+12) - 192 = 0$

☐ **Exercise 201**

a) Solve the following equation and give the solutions in simplest surd form. $t^2 + 10t + 5 = 0$

b) Solve the following equation and give the solutions correct to two decimal places
$-4t^2 - 40t - 20 = 0$

☐ **Exercise 202** Solve the equation $3x^3 + 5x^2 - 2x = 0$ by factorising. No other method may be used.

☐ **Exercise 203**
Solve the equation $6x^2 = 36x - 42$ and give the solutions in simplest surd form.

☐ **Exercise 204**
Solve the equation $x^2 + 2x + 5 = 0$

☐ **Exercise 205** Solve the following equations and give the solutions in simplest surd form.

a) $t^2 - 6t + 3 = 0$

b) $-7t^2 + 42t - 21 = 0$

☐ **Exercise 206** Solve for x the following equations

a) $3x^2 - 4x - 3 = 0$

b) $4x^2 - 12x + 16 = 0$

c) $\frac{2}{x} - 3 + x = 0$

d) $x^4 - 3x^2 - 108 = 0$

☐ **Exercise 207**
Solve the equation $40x^2 - 40x = -10$.

☐ **Exercise 208** What constant must be added to each expression in order to create a perfect square?

a) $x^2 - 4x$

b) $x^2 + 18x$

c) $y^2 + 3y$

d) $y^2 - 5y$

☐ **Exercise 209**
Solve the equation by completing the square on the left hand side : $x^2 - 2x = 1$

9.5 Quadratics and word problems

☐ **Exercise 210**
Is it possible to find a number x such that the triangle with sides x, $x+1$ and $x+2$ is right angled? If so, find all the possible solutions.

☐ **Exercise 211** Consider the continued fraction $x = \frac{5}{5+\frac{5}{5+\frac{5}{5+...}}}$.

a) Which of the following is this equivalent to?
  Ⓐ $x = \frac{5}{x}$
  Ⓑ $x = \frac{5}{5+x}$
  Ⓒ $5 = \frac{x}{5+x}$

b) Hence, find the exact value of the continued fraction x.

☐ **Exercise 212**
The continued surd $x = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+...}}}}$ has an exact value. Find the exact value of x in simplest surd form.
Hint : Express $\sqrt{1+x}$ in term of x.

9.6 Equations reducible to quadratics

☐ **Exercise 213**
Equations reducible to quadratics for beginners

a) Solve $X^2 - 13X + 36 = 0$.
b) Solve $x^4 - 13x^2 + 36 = 0$. *Hint* : Let $X = x^2$
c) Solve $\frac{1}{t^2} - 13 \times \frac{1}{t} + 36 = 0$. *Hint* : Let $X = \ldots$

☐ **Exercise 214**
Solve the equation $\frac{x}{x^2-9} + \frac{1}{x+3} = 1$

☐ **Exercise 215**
Solve $\frac{7x}{x-1} - \frac{2x+1}{x+1} = 2$

☐ **Exercise 216**
Solve $\frac{7x}{x+2} - \frac{2x+1}{x-2} = 2$

☐ **Exercise 217**
Solve the equation $\frac{u+3}{2u-7} = \frac{2u-1}{u-3}$.

☐ **Exercise 218**

a) Solve $\frac{2}{a+3} + \frac{a+3}{2} = \frac{10}{3}$
b) Hence or otherwise, solve $\frac{2}{a^2+3} + \frac{a^2+3}{2} = \frac{10}{3}$

☐ **Exercise 219**
Solve $\frac{\frac{1}{x}-\frac{2}{3}}{\frac{1}{x}+\frac{2}{3}} = \frac{x-2}{x+2}$

☐ **Exercise 220**
Solve $\frac{\frac{1}{x}-\frac{2}{3}}{\frac{1}{x}+\frac{2}{3}} = \frac{x+2}{x-2}$

☐ **Exercise 221** A rectangular area can be completely tiled with 200 square tiles. If the side length of each tile was increased by 1cm, it would only take 128 tiles to fill the area. Find the side length of each tile.

9.7 Quadratics and domain

☐ **Exercise 222** In the expression $\frac{1}{x^2-121}$, what value(s) are not allowable as substitutions for the variable x?

☐ **Exercise 223** In the expression $\frac{1}{50-2x^2}$, what value(s) are not allowable as substitutions for the variable x?

☐ **Exercise 224** In the expression $\frac{7}{\frac{1}{q}+10}$, what value(s) are not allowable as substitutions for the variable x?

☐ **Exercise 225** What is the natural domain of the function f defined by $f(x) = \frac{x^2+64}{x^2-64}$?

☐ **Exercise 226** What is the natural domain of the function f defined by $f(x) = \frac{x^2-64}{x^2+64}$?

9.8 Quadratic Identities

☐ **Exercise 227** Show that n^2 can be written $n^2 \equiv a(n+3)^2 + b(n+3) + c$ for some numbers a, b and c.

9.9 Quadratic Inequations

How to solve a Quadratic Inequation?
- First, Put everything on the Left Hand Side (LHS)
- Then, sketch the graph of the LHS. It is a parabola if the LHS is a polynomial of degree 2. You will need to factorise to find the x-intercepts if any.
- Finally, read the solution from the graph : The LHS is positive when the graph is above the x-axis and negative when the graph is below the x-axis.

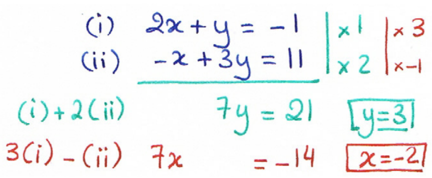
☐ **Exercise 228** Solve the inequation
$(x-3)(x+5) > 0$

☐ **Exercise 229** Solve the following inequations

a) $x^2 + 5x + 4 \leqslant 0$
b) $-x^2 - 5x - 4 \geqslant 0$
c) $-x^2 - 5x - 4 < 0$

10 Simultaneous equations

How to solve simultaneous equations?
If they are linear, use *elimination*! That is, multiply each equation by a well chosen numbers such that when you add, one of the variable disappears. And then do it again for the other variable.



☐ **Exercise 230**
Find the coordinates of point of intersection of the straight lines with respective equations $y = 2x - 3$ and $y = x + 1$.
Sketch the graph of the lines to check your answer.

☐ **Exercise 231**
Find the coordinates of point of intersection of the straight lines with respective equations $y = 2x - 3$ and $y = -5x + 11$.
Sketch the graph of the lines to check your answer.

☐ **Exercise 232**
Find the coordinates of point of intersection of the straight lines with respective equations $x + 3y = 7$ and $-x + y + 2 = 0$.
Sketch the graph of the lines to check your answer.

☐ **Exercise 233**
Solve by substitution $\begin{cases} 4x - 5y = 2 \\ x + 10y = 41 \end{cases}$

☐ **Exercise 234**
Solve by elimination $\begin{cases} 3x - 2y = 11 \\ 4x + 3y = 43 \end{cases}$

☐ **Exercise 235** Solve $\begin{cases} 5x - 3y = 28 \\ 2x - 3y = 22 \end{cases}$
Sketch the graph of the lines to check your answer.

☐ **Exercise 236** Solve $\begin{cases} 15x + 2y = 27 \\ 3x + 7y = 45 \end{cases}$

11 Numbers and Surds

☐ **Exercise 237** Decrease $1\frac{1}{2}$ by 20%.

☐ **Exercise 238** Write the recurring decimal $x = 1.\dot{2}\dot{3} = 1.23232323...$ as a fraction.

☐ **Exercise 239** Write the rational number $x = \frac{473}{6}$ as a recurring decimal.

Surds : Forbidden or allowed ?

- You *may not* 'break' a surd into two surds if you have a sum (or a difference) inside. For instance, $\sqrt{9+16} = 5$ which is not equal to $\sqrt{9} + \sqrt{16} = 7$
- On the other hand, you *may* 'break' a surd into two surds if you have a product (or a quotient) inside, i.e.
$\sqrt{ab} = \sqrt{a}\sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

TIP! Use $\sqrt{ab} = \sqrt{a}\sqrt{b}$ to simplify surds. For instance, $\sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$.

☐ **Exercise 240** [2015 Y11 2U Y]
Simplify $\sqrt{48} + 2\sqrt{75}$

☐ **Exercise 241** [2015 Y11 2U Y]
Simplify $\frac{\sqrt{22}}{\sqrt{176}}$ and rationalise the denominator.

12 Indices = exponents

12.1 [Y8] positive and zero indices, includes power of a power

☐ **Exercise 242**
Discovering the index Laws
Fill in the blanks

a) $x \times x = \ldots$
b) $x^2 \times x = \ldots$
c) $x^2 \times x^3 = \ldots$
d) $x^9 \times x^2 = \ldots$
e) $x^6 \times x^3 = \ldots$
f) $x^{16} \times x^{10} = \ldots$
g) *Challenge!* $x^{2017} \times x^{10} = \ldots$
h) *The rule* : For any numbers n and p, $x^n \times x^p = \ldots$

☐ **Exercise 243**
Discovering the index Laws
Fill in the blanks

a) $\frac{x^2}{x} = \ldots$
b) $\frac{y^3}{y} = \ldots$
c) $\frac{x^4}{x^2} = \ldots$
d) $\frac{x^5}{x^2} = \ldots$
e) $\frac{x^2}{x^2} = \ldots$
f) $\frac{x^{10}}{x^3} = \ldots$
g) $\frac{x^{25}}{x^8} = \ldots$
h) $\frac{x^5}{x^5} = \ldots$
i) *Challenge!* $\frac{x^{2017}}{x^8} = \ldots$
j) *Challenge!* $\frac{x^{35}}{x^{18}} = \ldots$

k) *The rule* : For any numbers n and p, with $n > p$ $\frac{x^n}{x^p} = \dots$

☐ **Exercise 244**
What is $(-1)^n$ *?*
Fill in the blanks

a) $(-1)^2 = \dots$
b) $(-1)^3 = \dots$
c) $(-1)^4 = \dots$
d) $(-1)^5 = \dots$
e) $(-1)^7 = \dots$
f) $(-1)^{10} = \dots$
g) $(-1)^{12} = \dots$
h) *Challenge!* $(-1)^{46} = \dots$
i) *Challenge!* $(-1)^{2017} = \dots$
j) *The rule* : For any integer n, with $n > 0$, $(-1)^n = \dots$

☐ **Exercise 245**
Discovering one more index Law
Fill in the blanks

a) $\frac{y^3}{y}$
b) $\frac{x^4}{x^2} = \dots$
c) $\frac{x^5}{x^2} = \dots$
d) $\frac{x^2}{x^2} = \dots$
e) $\frac{x^{10}}{x^3} = \dots$
f) $\frac{x^{25}}{x^8} = \dots$
g) $\frac{x^5}{x^5} = \dots$
h) *Challenge!* $\frac{x^{2017}}{x^8} = \dots$
i) *Challenge!* $\frac{x^{35}}{x^{18}} = \dots$
j) *The rule* : For any numbers n and p, with $n > p$ $\frac{x^n}{x^p} = \dots$

☐ **Exercise 246**
Investigation : What should x^0 be?

☐ **Exercise 247**
Discovering one more index Law
What should $(x^n)^p$ be?

Fill in the blanks

a) $\left(x^2\right)^2 = x^2 \times x^2 = x^{\square}$
b) $\left(x^3\right)^2 = x^3 \times x^3 = x^{\square}$
c) $\left(x^2\right)^3 = x^2 \times x^2 \times x^2 = x^{\square}$
d) $\left(x^3\right)^3 = \dots$
e) $\left(x^5\right)^2 = \dots$
f) $\left(x^7\right)^3 = \dots$
g) $\left(x^5\right)^8 = \dots$
h) $\left(x^6\right)^{12} = \dots$
i) *Challenge!* $\left(x^{2017}\right)^{10} = \dots$
j) *The rule* : For any numbers n and p, with $n > p$ $(x^n)^p = \dots$

☐ **Exercise 248**
■ *Discovering one more index Law, Power of a quotient*
What should $\left(\frac{a}{b}\right)^n$ be?

Fill in the blanks

a) $\left(\frac{a}{b}\right)^2 = \frac{a}{b} \times \frac{a}{b} = \frac{\dots\times\dots}{\dots\times\dots} = \frac{a^{\square}}{b^{\square}}$
b) $\left(\frac{a}{b}\right)^3 = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \frac{a^{\square}}{b^{\square}}$
c) $\left(\frac{a}{b}\right)^4 = \dots$
d) $\left(\frac{a}{b}\right)^5 = \dots$
e) $\left(\frac{a}{b}\right)^6 = \dots$
f) *Challenge!* $\left(\frac{2}{3}\right)^{2017} = \dots$
g) *The rule* : For any numbers a and b, and for any positive integer n, $\left(\frac{a}{b}\right)^n = \dots$
h) Is this rule also true when $n = 0$?

■ *Let's practise the "Power of a quotient" rule*
Simplify the following :

a) $\left(\frac{7}{6}\right)^2 = \dots$
b) $\left(\frac{x}{2}\right)^5 = \dots$
c) $\left(\frac{x}{3y}\right)^6 = \dots$
d) *Challenge!* $\left(\frac{5x}{2y}\right)^{11} \times 8y^3 = \dots$

☐ **Exercise 249**
Investigation : What about $(x+y)^n$? Can we find a nice formula for that?

☐ **Exercise 250** [Y8 Indices in a nutshell]
NO calculator! Give your answers in index form.

a) $2x^7 \times 5x^6$
b) $3x^4 + 5x^4$
c) $7x^0 - (7x)^0$
d) $5^4 \times 5^6$
e) $\left(7^4\right)^3$
f) $\left(11^4\right)^3 \times 11^6$
g) $(6m)^2$
h) $\frac{2^7}{2^3}$

☐ **Exercise 251** [Y8 Indices in a nutshell]
NO calculator! Give your answers in index form.

a) $3x^4 \times 5x^6$

b) $3x^4+5x^6$
c) $3x^0+(5x)^0$
d) $3^4\times 3^6$
e) $\left(5^4\right)^2$
f) $\left(3^4\right)^8\times 3^6$
g) Challenge! $\left(3^2\right)^4\times 9^5=3^{\square}$

☐ **Exercise 252** [More Y8 Indices]

a) $5x^7\times 7x^6$
b) 3^2+2^3
c) $\dfrac{\left(7^{12}\times 7^4\right)^0}{7^5}$
d) $\dfrac{7^{12}\times\left(7^4\right)^0}{7^5}$
e) $11^4\times 11^6$
f) $\left(7^4\right)^{10}$
g) $\left(\dfrac{7^2}{2}\right)^3$
h) $\left(\dfrac{7^2}{2}\right)^3\times 16=2^{\square}\times 7^{\square}$
i) $\left(2^3\right)^4\times 8^5=2^{\square}$
j) Simplify fully $\dfrac{10^7}{5^4}$

☐ **Exercise 253** [More (hard) Y8 Indices]

a) $5^7\times 5^6$
b) $2x^7\times 5x^{11}\times 3x^0$
c) $\dfrac{4^{17}}{4^{12}}$
d) $\dfrac{4^2+1^3}{4^0}$
e) $7x^2\times 3x^2$
f) $7x^2+3x^2$
g) $\dfrac{\left(3^8\times 3^4\right)^0}{3^5}$
h) $\dfrac{3^8\times\left(3^4\right)^0}{3^5}$
i) $13^4\times 13^6$
j) $\left(2^4\right)^6$
k) $\left(4^2\right)^6=2^{\square}$
l) $\left(\dfrac{5^7}{2}\right)^3$
m) Is it true that $5\times 2^3=10^3=1000$?
n) Prove without a calculator that $2^{100}-2^{99}=2^{99}$
o) Prove without a calculator that $5^{33}-5^{32}=11\times 5^{32}$

☐ **Exercise 254** Compare without a calculator $A=\dfrac{9^4\times 3^2}{3^7}$ and $B=5^2$.

12.2 [Y9] Negative indices and significant figures

☐ **Exercise 255** [NO CALCULATOR. Negative Indices]

a) $5^{-9}\times 5^6$
b) $2x^7\times -5x^{-11}\times 3x^0$
c) $\dfrac{4^{17}}{4^{-12}}$
d) $\dfrac{4^2+1^{-3}}{4^0}$
e) $7x^{-3}\times 3x^{12}$
f) $7x^{-2}+3x^{-2}$
g) $\dfrac{\left(3^{-8}\times 3^{-4}\right)^0}{3^{-5}}$
h) $\dfrac{3^2\times\left(3^4\right)^0}{3^{-5}}$
i) $13^4\times 13^{-8}$
j) $\left(2^4\right)^{-6}$
k) $\left(4^2\right)^{-7}=2^{\square}$
l) $\left(\dfrac{5^{-7}}{2}\right)^3$
m) (No calculator)Is it true that $5^{-3}\times 2^{-3}=0.001$?

12.3 [Y9/10/11] Fractional indices

☐ **Exercise 256** [NO CALCULATOR. Fractional Indices]

a) $25^{\frac{1}{2}}$
b) $8^{\frac{1}{3}}$
c) $8^{\frac{2}{3}}$
d) $1000^{\frac{2}{3}}$
e) $\left(81x^{12}\right)^{\frac{1}{2}}$
f) $\left(121x^4y^6\right)^{\frac{1}{2}}$
g) $\sqrt{49t^8}$
h) $\sqrt{x}\times x^{\frac{7}{2}}$
i) $\sqrt[3]{x}\times x^{\frac{5}{3}}$
j) $\sqrt{6x}\times(6x)^{\frac{3}{2}}$

☐ **Exercise 257** (*) Consider the numbers $A=(1-2\times 10^{-8})(1+2\times 10^{-8})$ and $B=(1-2\times 10^{-8})^2(1+2\times 10^{-8})^2$. The goal of this exercise is to compare them, that is to establish if $A>B$, or $B>A$ or $A=B$.

a) Evaluate A and B with your calculator and compare A and B.

b) i) Let $x = 10^{-8}$. Expand and simplify A and B after writing them in terms of x.

ii) Hence find the exact values of A and B.

iii) Compare this answer to your answer to question 1.

In case you were wondering, this was written using LaTeX *(free software for beautiful documents).*

ANSWERS (rather than full solutions)

Answer to Exercise 1 .

a) $3+5x+2+9x=14x+5$

b) $3\times 5x+2+9x=24x+2$

c) $3+5x\times 2+9x=3+19x$

d) $3\times 5x\times 2+9x=39x$

e) $3\times 5x\times 2\times 9x=270x^2$

Answer to Exercise 2 .

a) $-5x+2-4x-5=-9x-3$

b) $-5x\times 2-4x-5=-14x-5$

c) $-5x-4x(-5)=15x$

d) $-5x\times 2\times -4x=-200x^2$

e) $-5x+2\times -4x\times -5)=35x$

Answer to Exercise 3 .

a) $-3x+7y+8x-5y=5x+2y$

b) $-3x\times 7y+8x\times -5=-21xy+8x-5y$

c) $-3x\times 7y\times 8x\times -5=840x^2y^2$

Answer to Exercise 4 .

$5z^8+5+3xy+6z^8+3+yx=11z^8+4xy+8$

Answer to Exercise 5 .

$2xy^2-8x^2\cdot 3y+5y\left(-2xy\right)=-24x^2y-8xy^2$

Answer to Exercise 6 .

$\frac{-7x+4x}{-8x}=\frac{-3x}{-8x}=\frac{3}{8}$

Answer to Exercise 7 .

$7+ba$.

Answer to Exercise 8 .

$9+3x$.

Answer to Exercise 9 .

When $x=-2$, $3x^2-(2x+3)=3(-2)^2-(2\times(-2)+3)$
$=13$

Answer to Exercise 10 .

- When $x=-3$, $-x^2-(5x+4)$
  $=-(-3)^2-(5\times -3+4)=-9-(-11)=-9+11=2$
- When $x=-2$, $-x^2-(5x+4)$
  $=-(-2)^2-(5\times -2+4)=-4-(-6)=-4+6=2$

Charlotte is right!

Answer to Exercise 11 .

$-7(7a-4)+3=-49a+28+3=-49a+31$

Answer to Exercise 12 .

a) $2x+10+6(x+4)=8x+34$

b) $3x+8+2(6x+1)=15x+10$

Answer to Exercise 13 .

a) $x(x+7)+2(x+10)=x^2+9x+20$

b) $x(x+2)+5(7x+2)=x^2+37x+10$

Answer to Exercise 14 .

a) $7(n+1)-4(n+3)=7(n+1)+(-4)\times(n+3)=7(n+1)+(-4)\times n+(-4)\times 3=7n+7-4n-12=3n-5$

b) $7(w-6)-3(w-1)=4w-39$

Answer to Exercise 15 .

$3s+5-(2s+8)=3s+5-2s-8=s-3$

Answer to Exercise 16 .

$3(2x-5)-(7x+4)=6x-15-7x-4=-x-19$

Answer to Exercise 17 .

a) $-2(2x-5)-(2-5x)=-4x+10-(2-5x)=x+8$

b) $-2(2x-5)-(2-5x)=x+8$ for all values of x so, when $x=-12$, it is equal to $-12+8=-4$.

Answer to Exercise 18 .

$2x(5-x)-\left(5x^2+3x-7\right)$
$=10x-2x^2-\left(5x^2+3x-7\right)$
$=10x-2x^2-5x^2-3x+7=-7x^2+7x+7$

Answer to Exercise 19 .

$x-2y+3t-\left(4x+5y-6t\right)=x-2y+3t-4x-5y+6t=-3x-7y+9t$

Answer to Exercise 20 .

$-\left(4+2\left(x+8y\right)\right)+6\left(-2x+y\right)=-14x-10y-4$

Answer to Exercise 21 .

$-\left(6-2\left(x-7y\right)\right)+6\left(-3x+y\right)=-16x-8y-6$

Answer to Exercise 22 .

$3a(a-6)-\left(a^2-3a+7\right)=3a^2-18a-\left(a^2-3a+7\right)=3a^2-18a-a^2+3a-7=2a^2-15a-7$

Answer to Exercise 23 .

a) $A=(3x+2)(3x-5)-(-7-9x)=9x^2-9x-10-(-7-9x)=9x^2-9x-10+7+9x=9x^2-3$.

b) When $x=\frac{1}{3}$, $A=9\left(\frac{1}{3}\right)^2-3=9\times\frac{1}{9}-3=1-3=2$.

Answer to Exercise 24 .

$2a\left(2a^2-5a+10\right)-a^2(a+1)=3a^3-11a^2+20a$

Answer to Exercise 25 .

$2t(t-3)+3-t(t+6)==2t^2-6t+3-t(t+6)=2t^2-6t+3-t^2-6t=t^2-12t+3$

Answer to Exercise 26 .
$4y\left(2y^2-7y+6\right)-y^2\left(y+9\right)$
$=8y^3-28y^2+24y-y^2\left(y+9\right)$
$=8y^3-28y^2+24y-y^3-9y^2$
$=7y^3-37y^2+24y$

Answer to Exercise 27 .
$(x+2)(x+10)=x^2+10x+2x+20=x^2+12x+20$

Answer to Exercise 28 .
$(3x+2)(x-10)=3x^2-30x+2x-20=3x^2-28x-20$

Answer to Exercise 29 .
$(2m+6)(m-6)=2m^2-6m-36$

Answer to Exercise 30 .
$\left(7y-6\right)\left(6y+6\right)=42y^2+6y-36$

Answer to Exercise 31 .
$\left(5y-3\right)\left(-4y-1\right)=-20y^2+7y+3$

Answer to Exercise 32 .
$5(-3x-5)(5x+1)=-75x^2-140x-25$

Answer to Exercise 33 .
$2\left(y+8\right)\left(y+7\right)=2y^2+30y+112$

Answer to Exercise 34 .
$-8(-6x-9)(x+2)-168x=48x^2+144$

Answer to Exercise 35 .

a) $3x^2y(7y-9x)=21x^2y^2-27x^3y$.
b) $3x^2+y\left(7y-9x\right)=3x^2+7y^2-9xy$
c) $\left(3x^2+y\right)\left(7y-9x\right)=21x^2y-27x^3+7y^2-9xy$

Answer to Exercise 36 .

a) $(x+3)(2x-5)-10x=2x^2+x-15-10x$
$=2x^2-9x-15$.
b) $10x-(x+3)(2x-5)$ is the opposite of the expression in the previous question so we should get the same thing with all the signs reversed. $\therefore\ 10x-(x+3)(2x-5)=-2x^2+9x+15$
c) $10x-(x+3)(2x-5)$
$=10x-[(x+3)(2x-5)]$
$=10x-(2x^2+x-15)=10x-2x^2-x+15$
$=-2x^2+9x+15$, which is what we got in the previous question. All good!

Answer to Exercise 37 .
$2m^2-(2m+4)(m-2)=8$

Answer to Exercise 38 .

a) $(x+3)^2=x^2+6x+9$.
b) $(3x-5y)^2=9x^2-30xy+25y^2$.
c) $(3x-5y)(3x+5y)=9x^2-25y^2$.
d) $(x+\frac{1}{x})^2=x^2+2+\frac{1}{x^2}$.

Answer to Exercise 39 .

a) $(4y-5)^2=16y^2-40y+25$.
b) $(x+3)(x-3)=x^2-9$.
c) $(3m-7)(2m-4)-(m^2+15m-8)$
$=6m^2-12m-14m+28-(m^2+15m-8)$
$=6m^2-26m+28-m^2-15m-8$
$=5m^2-41m+20$.

Answer to Exercise 40 .

a) $(t+3)(t-3)=t^2-9$
b) $\left(10x-7y\right)\left(10x+7y\right)=100x^2-49y^2$
c) $\left(y+\frac{1}{4}\right)\left(y-\frac{1}{4}\right)=y^2-\frac{1}{16}$.
d) $\left(7x-\frac{5}{3}\right)\left(7x+\frac{5}{3}\right)=49x^2-\frac{25}{9}$.

Answer to Exercise 41 .

a) $T=-22x+12$.
b) When $x=\frac{4}{11}$, $T=4$.

Answer to Exercise 42 .
$(x+12)(x-2)=x^2+10x-24$

Answer to Exercise 43 .
$(x+3)^2=x^2+6x+9$, NOT x^2+9 (test for $x=1$). [Perfect Square formula or FOIL]

Answer to Exercise 44 .
$(x-3)^2=x^2-6x+9$, NOT x^2+9 (test for $x=1$). [Perfect Square formula or FOIL]

Answer to Exercise 45 .
$(x-9)^2=x^2-18x+81$, NOT x^2+81 (test for $x=1$). [Perfect Square formula or FOIL]

Answer to Exercise 46 .
$(3x+5)^2=9x^2+30x+25$. [Perfect Square formula or FOIL]

Answer to Exercise 47 .
$(3x-4y)^2=9x^2-24xy+16y^2$. [Perfect Square formula or FOIL]

Answer to Exercise 48 .
$(7x+t)^2=49x^2+14xt+t^2$. [Perfect Square formula or FOIL]

Answer to Exercise 49 .
$(3x-4x)^2=(-x)^2=x^2$.

Answer to Exercise 50 .
$R=-3x+7$ so with $x=100000$, $99997^2-99999\times 99998=-299993$.

Answer to Exercise 51 .
Each side of the new square $= 3(x+3) = 3x+9cm$.
Area of the new square $= (3x+9)^2 = 9x^2+36x+81cm^2$.

Answer to Exercise 52 .
Each side of the new square $= 5(u-4) = 5u-20cm$.
Area of the new square $= (5u-20)^2 = 25u^2 - 200u + 400cm^2$.

Answer to Exercise 53 .
Height of the box= 2 centimetre
Width $= 7y$ centimetres
Length $= 2 \times 7y + 2 = 14y + 2$.
Volume $2 \times 7y \times (14y+2)196y^2 + 28y$.

Answer to Exercise 54 .
$6t^3s - 64s^2t^2 = 2t^2s(3t-32s)$

Answer to Exercise 55 .
$\frac{3x}{7} + \frac{2y}{7} = \frac{1}{7}(3x+2y)$

Answer to Exercise 56 .

a) $x^2 - 7x = x(x-7)$
b) $32(at)^4 - 48a^7t^3 = 16 \times 2a^4t^4 - 16 \times 3a^7t^3$
$= 16a^4t^3(2t-3a^3)$

Answer to Exercise 57 .

a) $x^2 - 9 = (x-3)(x+3)$
b) $x^2+6x+8 = (x+4)(x+2)$. Use PSF with $P = 8, S = 6$ and $F = 4, 2$.
c) $6x^2 + 13x - 5 = (3x-1)(2x+5)$. Use PSF with $P = -30, S = 13$ and $F = 15, -2$.

Answer to Exercise 58 .

a) $7x^2 - 63 = 7(x^2 - 9) = 7(x-3)(x+3)$
b) $11x^2 - 11x - 22 = 11(x^2 - x - 2) = 11(x+1)(x-2)$
c) $x^4 - 16 = (x^2-4)(x^2+4) = (x-2)(x+2)(x^2+4)$.

Answer to Exercise 59 .
$S = 7x(7x-6)$ so when $x = \dfrac{1}{7}$, $S = 1(1-6) = -5$.

Answer to Exercise 60 .

a) $\frac{35}{45} = \frac{7}{9}$
b) $\frac{12}{18} = \frac{2}{3}$
c) $\frac{88}{99} = \frac{11\times8}{11\times9} = \frac{8}{9}$
d) $\frac{3\times5^3}{7\times5^2} = \frac{15}{7}$
e) $\frac{3\times7^4}{7^2\times12} = \frac{49}{4}$
f) $\frac{3\times x^3}{7\times x^2} = \frac{3x^3}{7x^2} = \frac{3x}{7}$
g) $\frac{3y^4}{12y^2} = \frac{y^2}{4}$

Answer to Exercise 61 .

Answer to Exercise 62 .
$\frac{4t-8}{2} = \frac{2(2t-4)}{2} = 2t-4$

Answer to Exercise 63 .
$\frac{30t^7}{15t^3-40t} = \frac{30t^7}{5t(3t^2-8)} = \frac{6t^6}{3t^2-8}$

Answer to Exercise 64 .
$\frac{3x-21}{6} = \frac{3(x-7)}{3\times2} = \frac{x-7}{2}$

Answer to Exercise 65 .
$\frac{12}{3x-21} = \frac{3\times4}{3(x-7)} = \frac{4}{x-7}$

Answer to Exercise 66 .
$\frac{3x-21}{2x-14} = \frac{3(x-7)}{2(x-7)} = \frac{3}{2}$

Answer to Exercise 67 .
$\frac{y+8}{y^2-64} = \frac{y+8}{(y+8)(y-8)} = \frac{1}{y-8}$

Answer to Exercise 68 .
$\frac{a^2-1}{a+1} = \frac{(a+1)(a-1)}{a+1} = a-1$

Answer to Exercise 69 .
$\frac{a^2-49}{a-7} = \frac{(a-7)(a+7)}{a-7} = a+7$

Answer to Exercise 70 .
$\frac{2x^2-32}{x-4} = \frac{2(x^2-16)}{x-4} = \frac{2(x-4)(x+4)}{x-4} = 2(x+4)$

Answer to Exercise 71 .
$\frac{x-5}{2x^2-7x-15} = \frac{x-5}{(2x+3)(x-5)} = \frac{1}{2x+3}$

Answer to Exercise 72 .
$\frac{x^2-25}{2x^2-7x-15} = \frac{(x-5)(x+5)}{(2x+3)(x-5)} = \frac{x-5}{2x+3}$

Answer to Exercise 73 .
$\frac{3(7x-6)-8x+5}{26} = \frac{13x-13}{26} = \frac{13(x-1)}{13\times2} = \frac{x-1}{2}$

Answer to Exercise 74 .
$\frac{3(7x-6)-(17x+10)}{2} = \frac{4x-28}{2} = \frac{4(x-7)}{2} = 2(x-7)$

Answer to Exercise 75 .
$\frac{x^2-5x+7}{-5x+7+x^2} - 3 = 1-3 = -2$

Answer to Exercise 76 .
$\frac{x^2+5x-3}{-6+2x^2+10x} = \frac{x^2+5x-3}{2(x^2+5x-3)} = \frac{1}{2}$

Answer to Exercise 77 .
$\frac{x^2+5x-3}{-3+x^2+5x} = \frac{x^2+5x-3}{x^2+5x-3} = 1$

Answer to Exercise 78 .
$\frac{x-3}{3-x} = \frac{x-3}{-(x-3)} = -1$

Answer to Exercise 79 .
$$\frac{a^2b^4}{6} \times \frac{9}{a^2b^2} = \frac{a^2b^4\times9}{6a^2b^2} = \frac{3b^2}{2}$$

Answer to Exercise 80 .
$21p^3q^7 \times \boxed{5p^4q^{13}} = 105p^7q^{20}$

Answer to Exercise 81 .
$\frac{72x^5y}{21} \div \frac{x^2}{14y^2} = \frac{72x^5y\times14y^2}{21x^2} = 48x^3y^3$

Answer to Exercise 82 .

Rewrite the question : $\frac{20p^3q^7}{\boxed{}} = 5p^2q^5$

Fill in the box : $\frac{20p^3q^7}{\boxed{4pq^2}} = 5p^2q^5$

Answer to Exercise 83 .

$\frac{3x^2}{8y^5} \div \frac{15x^3}{4y} = \frac{3x^2}{8y^5} \times \frac{4y}{15x^3} = \frac{3x^2 \times 4y}{8y^5 \times 15x^3} = \frac{3 \times 4 \times x^2 y}{2 \times 4 \times 3 \times 5 \times y^5 x^3} = \frac{1}{10xy^4}$

Answer to Exercise 84 .

$\frac{12x^7}{8(xy)^5} \div \frac{15x^3}{45y} = \frac{12x^7 \times 45y}{8(xy)^5 \times 15x^3} = \frac{12 \times 45x^7 y}{8 \times 15x^5 y^5 x^3} = \frac{9}{2xy^4}$

Answer to Exercise 85 .

$\frac{3x+3}{x^2-2x} \times \frac{x^2+x-6}{x^2-1} = \frac{3(x+1)(x+3)(x-2)}{x(x-2)(x^2-1)} = \frac{3(x+3)}{x(x-1)}$

Answer to Exercise 86 .

a) $\frac{2a+8}{a+4} = \frac{2(a+4)}{a+4} = 2$

b) $\frac{x+2}{x-3} \times \frac{x^2-5x+6}{x^2-4} = \frac{x+2}{x-3} \times \frac{(x-2)(x-3)}{(x-2)(x+2)} = 1$. Use PSF for $x^2 - 5x + 6$ with $P = 6, S = -5$ and $F = -2, -3$ and Difference of Squares for $x^2 - 4$.

Answer to Exercise 87 .

[From Y9 programme]

$\frac{3m-6}{4} \times \frac{8m}{m^2-2m} = \frac{(3m-6)\times 8m}{4\times(m^2-2m)}$ [Get one big fraction]

$= \frac{3(m-2)\times 4\times 2\times m}{4m\times(m-2)}$ [Factorise numerator and denominator]

= 6 [Cancel the common factors]

Answer to Exercise 88 .

$\frac{3x+3}{x^2-2x} \times \frac{x^2+x-6}{x^2-1} = \frac{3(x+1)(x+3)(x-2)}{x(x-2)(x^2-1)} = \frac{3(x+3)}{x(x-1)}$

Answer to Exercise 89 .

$\frac{3x+3}{x^2-2x} \times \frac{x^2+x-6}{x^2-1} = \frac{3(x+1)(x+3)(x-2)}{x(x-2)(x^2-1)} = \frac{3(x+3)}{x(x-1)}$

Answer to Exercise 90 .

$\frac{3x+3}{x^2-2x} \div \frac{x^2-1}{x^2+x-6} = \frac{3x+3}{x^2-2x} \times \frac{x^2+x-6}{x^2-1} = \frac{3(x+1)(x+3)(x-2)}{x(x-2)(x^2-1)} = \frac{3(x+3)}{x(x-1)}$

Answer to Exercise 91 .

$\frac{-3xy}{3xy} - \frac{5}{3} = -1 - \frac{5}{3} = -\frac{3}{3} - \frac{5}{3} = -\frac{8}{3}$

Answer to Exercise 92 .

$\frac{8x+9}{6} + \frac{7x-4}{6} = \frac{8x+9+(7x-4)}{6} = \frac{8x+9+7x-4}{6} = \frac{15x+5}{6}$

Answer to Exercise 93 .

$\frac{2p^2+7p+6}{p+8} - \frac{3p^2+4p+5}{p+8} = \frac{3p-p^2+1}{p+8}$

Answer to Exercise 94 .

$\frac{3y^2+7y+5}{7} - \frac{6y^2+3}{7} = \frac{3y^2+7y+5-(6y^2+3)}{7} = \frac{7y-3y^2+2}{7}$

Answer to Exercise 95 .

$\frac{8v+5}{v-4} - \frac{5v+5}{v-4} = \frac{8v+5-(5v+5)}{v-4} = \frac{3v}{v-4}$

Answer to Exercise 96 .

$\frac{-9x-6\times 5x}{-10x} = \frac{-39x}{-10x} = 3.9$

Answer to Exercise 97 .

$\frac{3p^2+4p+3}{p-9} - \frac{2p^2+6p+6}{p-9} = \frac{p^2-2p-3}{p-9}$

Answer to Exercise 98 .

$\frac{3p^2+9}{p+5} - \frac{4p^2+5p}{p+5} = \frac{3p^2+9-(4p^2+5p)}{p+5} = \frac{-p^2-5p+9}{p+5}$

Answer to Exercise 99 .

$\frac{4n^2+7n+3}{n^2+n+8} - \frac{9n^2+5n+5}{n^2+n+8} = \frac{2n-5n^2-2}{n^2+n+8}$

Answer to Exercise 100 .

The lowest common denominator of the fractions $\frac{1}{5}$ and $\frac{1}{4}$ is 20

Answer to Exercise 101 .

The lowest common denominator of the fractions $\frac{1}{12}$ and $\frac{1}{60}$ is 60

Answer to Exercise 102 .

The lowest common denominator of the fractions $\frac{1}{12}$ and $\frac{1}{30}$ is 60

Answer to Exercise 103 .

The lowest common denominator of the fractions $\frac{1}{12y}$ and $\frac{7x}{y^2}$ is $12y^2$

Answer to Exercise 104 .

The correct explanation is Ⓒ : Sophie said that the result has to end up with identical denominators. The lowest product that contains all of the factors is $(x-2)(x+8)^2$, so that will be the lowest common denominator.

Answer to Exercise 105 .

a) For the fractions $\frac{2}{x}$and $\frac{7}{x^2}$, the lowest common denominator would be x^2. TRUE.

b) For the fractions $\frac{3}{x^2(x+2)}$ and $\frac{7}{(x+2)^2}$, the lowest common denominator would be $x(x+2)$. FALSE. (it is $x^2(x+2)^2$.)

c) For the fractions $\frac{4}{(x+6)^2(x-4)}$ and $\frac{7}{(x+6)^3}$, the lowest common denominator would be $(x+6)^3(x-4)$. TRUE.

Answer to Exercise 106 .

a) For the fractions $\frac{5}{x^3}$ and $\frac{1}{x^2}$, the lowest common denominator would be x^2. FALSE, it is x^3.

b) For the fractions $\frac{3}{(x+5)^2(x-3)}$ and $\frac{2}{(x+5)^3}$, the lowest common denominator would be $(x+5)^3(x-3)$. TRUE.

c) For the fractions $\frac{3}{x^2(x+2)}$ and $\frac{7}{(x+2)^2}$, the lowest common denominator would be $x^2(x+2)$. FALSE. (it is $x^2(x+2)^2$.)

Answer to Exercise 107 .

The lowest common denominator of the fractions $\frac{1}{12y^4z^2}$ and $\frac{1}{18y^2z^3}$ is $36y^4z^3$

Answer to Exercise 108 .
The lowest common denominator of the fractions $\frac{1}{9x^2z^5}$ and $\frac{1}{21x^3z^3}$ is $63x^3z^5$

Answer to Exercise 109 .
$\frac{4}{9xz} + \frac{1}{36xy} = \frac{16y+z}{36xzy}$

Answer to Exercise 110 .
$\frac{2x+1}{45} + \frac{3x+5}{30} = \frac{2(2x+1)+3(3x+5)}{90} = \frac{13x+17}{90}$

Answer to Exercise 111 .
$\frac{7p-q}{7} - \frac{p+6q}{5} = \frac{(7p-q)\times 5}{35} - \frac{(p+6q)\times 7}{35} = \frac{5(7p-q)-7(p+6q)}{35}$
$= \frac{28p-47q}{35}$

Answer to Exercise 112 .
$\frac{a+7}{a^2b} + \frac{b-5}{ab^2} = \frac{(a+7)b}{a^2b^2} + \frac{(b-5)a}{a^2b^2} = \frac{b(a+7)+a(b-5)}{a^2b^2} = \frac{ab+7b+ab-5a}{a^2b^2} = \frac{-5a+2ab+7b}{a^2b^2}$.

Answer to Exercise 113 .

a) $\frac{x}{2} - \frac{x}{5} = \frac{5x}{10} - \frac{2x}{10} = \frac{3x}{10}$.
b) $\frac{1}{x+1} + \frac{1}{x-1} = \frac{x-1}{(x+1)(x-1)} + \frac{x+1}{(x+1)(x-1)}$
$= \frac{2x}{(x+1)(x-1)}$.

Answer to Exercise 114 .
$\frac{-3x^2}{6x} - \frac{5x}{2} = -\frac{x}{2} - \frac{5x}{2} = -3x$

Answer to Exercise 115 .
$\frac{2}{5p} - \frac{4}{p} = \frac{2}{5p} - \frac{20}{5p} = -\frac{18}{5p}$

Answer to Exercise 116 .
$\frac{5x}{9(x+1)} - \frac{5}{x+1} = \frac{5x-45}{9(x+1)}$

Answer to Exercise 117 .
$\frac{4x}{5(x+4)} - \frac{4}{x+4} = \frac{4x-20}{5(x+4)}$

Answer to Exercise 118 .
$\frac{6x}{11(x+5)} - \frac{3}{x+5} = \frac{6x-33}{11}(x+5)$

Answer to Exercise 119 .
$\frac{1}{6} + \frac{6}{x-6} = \frac{x-6}{6(x-6)} + \frac{6\times 6}{6(x-6)} = \frac{x+30}{6(x-6)}$

Answer to Exercise 120 .
$\frac{a+2}{a^7b^5} - \frac{b^4+3}{a^6b^9} = \frac{b^4(a+2)-a(b^4+3)}{a^7b^9} = \frac{-3a+2b^4}{a^7b^9}$.

Answer to Exercise 121 .
$\frac{a}{4} + \frac{2a+1}{3} - a = \frac{3a}{12} + \frac{8a+4}{12} - \frac{12a}{12} = \frac{3a+8a+4-12a}{12} = \frac{-a+4}{12}$.

Answer to Exercise 122 .
$\frac{a}{4} - \frac{2a+1}{3} + a = \frac{3a}{12} - \frac{8a+4}{12} + \frac{12a}{12} = \frac{3a-8a-4+12a}{12} = \frac{7a-4}{12}$.

Answer to Exercise 123 .
$\frac{3}{x+7} - \frac{1}{x-7} = \frac{2x-28}{(x+7)(x-7)} = \frac{2x-28}{x^2-49}$

Answer to Exercise 124 .
[From Y9 5.3 Programme]
$\frac{4}{x^2+x} - \frac{3}{x^2-1}$
$= \frac{4}{x(x+1)} - \frac{3}{(x+1)(x-1)}$
$= \frac{4(x-1)}{x(x+1)(x-1)} - \frac{3x}{x(x+1)(x-1)}$
$= \frac{4(x-1)-3x}{x(x+1)(x-1)} = \frac{x-4}{x(x+1)(x-1)}$

Answer to Exercise 125 .
[From Y9 5.3 Programme]
$\frac{4}{x^2-9} + \frac{2}{3x+9} = \frac{4}{(x+3)(x-3)} + \frac{2}{3(x+3)}$
$= \frac{4\times 3}{3(x+3)(x-3)} + \frac{2(x-3)}{3(x+3)(x-3)} = \frac{12+2(x-3)}{3(x+3)(x-3)}$
$= \frac{2x+6}{3(x+3)(x-3)} = \frac{2(x+3)}{3(x+3)(x-3)} = \frac{2}{3(x-3)}$.

Answer to Exercise 126 .
$\frac{x-3}{x^2-16} - \frac{1}{x+4} = \frac{x-3}{(x+4)(x-4)} - \frac{(x-4)}{(x+4)(x-4)} = \frac{1}{x^2-16}$

Answer to Exercise 127 .
$\frac{9}{2y+6} + \frac{3}{4y-10} = \frac{9}{2(y+3)} + \frac{3}{2(2y-5)}$
$= \frac{9(2y-5)}{2(y+3)(2y-5)} + \frac{3(y+3)}{2(y+3)(2y-5)} = \frac{21y-36}{2(y+3)(2y-5)}$

Answer to Exercise 128 .
$\frac{x}{x^2-49} - \frac{7}{x+7} = \frac{x}{(x+7)(x-7)} - \frac{7(x-7)}{(x+7)(x-7)} = \frac{-6x+49}{(x+7)(x-7)}$

Answer to Exercise 129 .
$\frac{2x-25}{x^2-81} - \frac{3}{x+9} = \frac{2x-25}{(x+9)(x-9)} - \frac{3(x-9)}{(x+9)(x-9)} = \frac{-x+2}{(x+9)(x-9)}$

Answer to Exercise 130 .
$\frac{5}{x+5} - \frac{3}{(x-2)(x-9)} = \frac{5x^2-58x+75}{(x+5)(x-9)(x-2)}$

Answer to Exercise 131 .
$\frac{5}{(x-1)(x-9)} + \frac{2}{(x-1)(x+7)} = \frac{5(x+7)}{(x+7)(x-9)(x-1)} + \frac{2(x-9)}{(x+7)(x-9)(x-1)} = \frac{7x+17}{(x+7)(x-9)(x-1)}$

Answer to Exercise 132 .
$\frac{8}{3x+24} + \frac{3}{x^2+8x} = \frac{8}{3(x+8)} + \frac{3}{x(x+8)} = \frac{8x}{3x(x+8)} + \frac{3\times 3}{3x(x+8)} = \frac{8x+9}{3x(x+8)}$

Answer to Exercise 133 .
We factorise the first denominator using PSF and we get : $\frac{x+9}{x^2+14x+45} + \frac{x-8}{(x+5)(x+8)} = \frac{x+9}{(x+9)(x+5)} + \frac{x-8}{(x+5)(x+8)} = \frac{1}{x+5} + \frac{x-8}{(x+5)(x+8)} = \frac{x+8}{(x+5)(x+8)} + \frac{x-8}{(x+5)(x+8)} = \frac{2x}{(x+5)(x+8)}$

Answer to Exercise 134 .
We factorise the first denominator using PSF and we get : $\frac{x+7}{x^2+18x+77} + \frac{x-10}{(x+11)(x+10)}$
$= \frac{x+7}{(x+7)(x+11)} + \frac{x-10}{(x+11)(x+10)} = \frac{1}{x+11} + \frac{x-10}{(x+11)(x+10)}$
$= \frac{x+10}{(x+11)(x+10)} + \frac{x-10}{(x+11)(x+10)} = \frac{2x}{(x+11)(x+10)}$

Answer to Exercise 135 .
We factorise the first denominator using PSF and we get : $\frac{x+7}{x^2+18x+77} + \frac{x-10}{(x+11)(x+10)}$
$= \frac{x+7}{(x+7)(x+11)} + \frac{x-10}{(x+11)(x+10)} = \frac{1}{x+11} + \frac{x-10}{(x+11)(x+10)}$
$= \frac{x+10}{(x+11)(x+10)} + \frac{x-10}{(x+11)(x+10)} = \frac{2x}{(x+11)(x+10)}$

Answer to Exercise 136 .
We factorise the denominators using PSF and difference of squares respectively to get :
$\frac{x^2+10x+21}{x^2-49} - \frac{x+5}{x^2-2x-35} = \frac{(x+3)(x+7)}{(x-7)(x+7)} - \frac{x+5}{(x+5)(x-7)} = \frac{x+3}{x-7} - \frac{1}{x-7} = \frac{x+2}{x-7}$

Answer to Exercise 137 .

We factorise the denominators using PSF and difference of squares respectively to get : $\frac{x^2+10x+21}{x^2-49} - \frac{x+5}{x^2-2x-35} = \frac{(x+3)(x+7)}{(x-7)(x+7)} - \frac{x+5}{(x+5)(x-7)} = \frac{x+3}{x-7} - \frac{1}{x-7} = \frac{x+2}{x-7}$

Answer to Exercise 138 .

We factorise the denominators using PSF and difference of squares respectively to get : $\frac{1}{x^2-4x-32} - \frac{3}{x^2-64} = \frac{1}{(x+4)(x-8)} - \frac{3}{(x+8)(x-8)} = \frac{x+8}{(x+4)(x+8)(x-8)} - \frac{3(x+4)}{(x+4)(x+8)(x-8)} = \frac{-2x-4}{(x+4)(x+8)(x-8)}$

Answer to Exercise 139 .

$\frac{2}{x^2+7x-18} + \frac{3}{x^2-6x+8} = \frac{2}{(x+9)(x-2)} + \frac{3}{(x-2)(x-4)} = \frac{5x+19}{(x+9)(x-4)(x-2)}$

Answer to Exercise 140 .

Dividing by zero is not allowed, so the value $x = 0$ is not allowable as substitutions for the variable x.

Answer to Exercise 141 .

Dividing by zero is not allowed, so the value $x = -3$ (solution of $x+3 = 0$) is not allowable as substitutions for the variable x.

Answer to Exercise 142 .

Multiply the numerator and the denominator of the "main" fraction by ab.

$\frac{1}{\frac{1}{a}+\frac{1}{b}} = \frac{ab}{b+a}$

Answer to Exercise 143 .

Multiply the numerator and the denominator of the "main" fraction by ab.

$\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}} = \frac{b+a}{b-a}$

Answer to Exercise 144 .

Multiply the numerator and the denominator of the "main" fraction by ab.

$\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}} = \frac{b+a}{b-a}$.

Also $\frac{b+a}{a-b} = -\frac{b+a}{b-a}$ so our initial expression is equal to $\frac{b+a}{b-a} - \frac{b+a}{b-a} = 0$ and therefore does not depend on the values chosen for a and b.

Answer to Exercise 145 .

$-3x+5 = 12+4x \iff -7x = 7 \iff x = -1$

Answer to Exercise 146 .

a) $x+4 = 12$

b) $x+12 = 4$

c) $3x = 741$

d) $\frac{x}{3} = 18$

e) $9x = 72$

f) $9x = -36$

g) $10x = 3$

h) $x+6 = 2.5$

i) $3x = -4$

j) $3x+1 = 5.5$

Answer to Exercise 147 .

You may use a flow chart or the "onion layers" to know which operation was performed last : This is the operation you must undo first.

$-3x+5 = 12$

$\iff -3x = 12-5$

$\iff -3x = 7$

$\iff x = \frac{7}{-3} = -\frac{7}{3}$

Answer to Exercise 148 .

You may use flow charts.

a) $x-7 = 15$
$\Leftrightarrow x = 15+7$
$\Leftrightarrow x = 22$

b) $2x-7 = 15$
$\Leftrightarrow 2x = 15+7$
$\Leftrightarrow 2x = 22$
$\Leftrightarrow x = 11$

c) $7-2x = 15$
$\Leftrightarrow -2x = 15-7$
$\Leftrightarrow -2x = 8$
$\Leftrightarrow x = \frac{8}{-2} = -4.$

d) $\frac{x}{7} = 5$
$\Leftrightarrow x = 7\times 5$
$\Leftrightarrow x = 35$

e) $\frac{2x}{7} = 5$
$\Leftrightarrow 2x = 35$
$\Leftrightarrow x = \frac{35}{2} = 17.5$

Answer to Exercise 149 .

Teacher to check.

Answer to Exercise 150 .

Use substitution to check.

a) No

b) Yes

Answer to Exercise 151 .

Use substitution to check.

a) $x = -5$ is a solution of this equation.

b) $x = 3$ is not a solution of this equation.

c) $x = \frac{3}{2}$ is a solution of this equation.

Answer to Exercise 152 .

$\frac{5x}{2} - 2 = 3$

$\frac{5x}{2} = 5$

$5x = 10$

$x = 2.$

Answer to Exercise 153 .
$-3x+4-5x+8=12x \iff -8x+12=12x \iff 20x=12 \iff x=\frac{12}{20} \iff x=\frac{3}{5}$

Answer to Exercise 154 .
$-x+8=-11 \iff -x=-19 \iff x=19$

Answer to Exercise 155 .

a) $3x+4=10 \iff 3x=6 \iff x=2$
b) $9x-4=2(2x+8) \iff 9x-4=4x+16 \iff 5x=20 \iff x=4$
c) Multiply both sides by 8 to get $x-56=48$ i.e. $x=104$.

Answer to Exercise 156 .
If we expand and simplify, the x^2's cancel so this is really a first degree equation.Expand, simplify then solve as usual. $6x(x-2)-2x(3x-7)=14 \iff 2x=14$ so $x=7$.

Answer to Exercise 157 .
$-3(2x+5)+7(3x+10)=10x \iff 15x+55=10x \iff x=-11$

Answer to Exercise 158 .
Divide both sides by 100, get $x+8=9x-7 \therefore x=\frac{15}{8}$

Answer to Exercise 159 .
Divide both sides by 30, get $2x+3=x-5 \therefore x=-8$

Answer to Exercise 160 .
Divide both sides by 4, get $x+5=4x-10$ so $x=5$.

Answer to Exercise 161 .
Multiply both sides by 4, get $9x-15+8=8x$ so $x=7$.

Answer to Exercise 162 .
Multiply both sides by 5, get $4x-1+30x=10x+14$ so $x=\frac{5}{8}$.

Answer to Exercise 163 .
Multiply both sides by 3, get $2x+1=6x+1$ so $x=0$.

Answer to Exercise 164 .
Multiply both sides by 6 to get rid of the denominators : $\frac{x+1}{3}+\frac{5x-2}{6}=x+\frac{7}{6}$
$\iff 2(x+1)+5x-2=6x+7$
$\iff 7x=6x+7 \iff x=7$

Answer to Exercise 165 .
Multiply both sides by 14 to get rid of the denominators : $\frac{-70x-3}{7}+\frac{5x+2}{2}=8-x$
$\iff 2(-70x-3)+7(5x+2)=112-14x$
$\iff -105x+8=112-14x \iff x=-\frac{8}{7}$

Answer to Exercise 166 .

a) $16x-32=160+48x \Leftrightarrow x=-6$
b) $100(2x+3)=500-400x \Leftrightarrow x=\frac{1}{3}$
c) $8(12-20x)=40x+32 \Leftrightarrow x=\frac{8}{25}$

Answer to Exercise 167 .
Multiply both sides by 2 to get rid of the denominators : $\frac{x-13}{2}-\frac{7x-3}{2}=3x-\frac{7}{2} \iff x-13-(7x-3)=6x-7 \iff -6x-10=6x-7 \iff x=-\frac{1}{4}$

Answer to Exercise 168 .
Multiply both sides by 6 to get rid of the denominators : $\frac{x+1}{3}+\frac{5x-2}{6}=x+\frac{7}{6}$
$\iff 2(x+1)-(5x-2)=6x+7$
$\iff -3x+4=6x+7 \iff x=-\frac{1}{3}$

Answer to Exercise 169 .
If we expand and simplify, the x^2's cancel so this is really a first degree equation.Expand, simplify then solve as usual. $(3x-5)^2-(3x+5)^2=14 \iff -60x=14 \iff x=-\frac{14}{60}=-\frac{7}{30}$

Answer to Exercise 170 .

a) $x-4=3x+8 \iff 2x=-12 \iff x=-6$
b) $3(x+5)=x \iff 3x+15=x \iff x=-\frac{15}{2}=-7\frac{1}{2}$
c) Multiply both sides by 6 : $\frac{x+8}{3}=\frac{x-2}{2} \iff 2(x+8) \iff 2(x+8)=3(x-2) \iff 2x=3x-22 \iff x=22$.
d) $9x^2=81 \iff x^2=9 \iff x=\pm 3$.

Answer to Exercise 171 .
$-3(2x-7)-(3x+10)=10-x \iff -9x+11=10x \iff x=\frac{1}{8}$

Answer to Exercise 172 .
Multiply both sides by 2 to get rid of the denominators. $\frac{x-7}{2}+\frac{5x-3}{2}=3x-\frac{11}{2} \Leftrightarrow x-7+5x-3=8x-11$
$\Leftrightarrow 6x-10=8x-11 \Leftrightarrow 2x=1 \Leftrightarrow x=\frac{1}{2}$.

Answer to Exercise 173 .
Multiply both sides by 2 to get rid of the denominators. $\frac{x-7}{2}+\frac{5x-3}{2}=3x-\frac{11}{2} \iff 6x=6x-1$.
No solution

Answer to Exercise 174 .

a) $\frac{3}{x-2}=\frac{4}{2x-5}$ [Cross-Multiply]
$\iff 3(2x-5)=4(x-2) \iff x=\frac{7}{2}$
b) Let $u=\sqrt{x}$.
By (1.), $u=\frac{7}{2}$. Therefore, $\sqrt{x}=\frac{7}{2}$ so $x=\frac{49}{4}$.

Answer to Exercise 175 .
DRAW the rectangle and put all the information on your diagram.
$Perimeter=2(3y+2)+2(5y+7)=128+5y$
$2(3y+2)+2(5y+7)=128+5y \Leftrightarrow 16y+18=128+5y \Leftrightarrow 16y+18-18=128+5y-18 \Leftrightarrow 16y=5y+110 \Leftrightarrow 11y=110 \Leftrightarrow y=10$
The rectangle is 32 cm by 57 cm.

Answer to Exercise 176 .
$B = 0.17(180000 - B) \iff 100B = 17(180000 - B)$
$\iff 100B = 3060000 - 17B \iff 117B = 3060000$
$\iff B = \frac{3060000}{117} = \frac{340000}{13} \approx \$26153.84615\ldots$
The bonus to be shared between the employees of the company \$26153.85

Answer to Exercise 177 .
$\frac{1+n}{4+n} = \frac{2}{3} \iff 3(1+n) = 2(4+n) \iff n = 5.$

Answer to Exercise 178 .
Multiply both sides by $15x$, we get $x = -\dfrac{15}{2}$

Answer to Exercise 179 .
$2(x-5)+7 > 5x-2 \iff 2x-3 > 5x-2 \iff 2x > 5x+1 \iff -1 > 3x \iff 3x < -1 \iff x < -\frac{1}{3}$

Answer to Exercise 180 .
$x^2 = 25 \iff x = \sqrt{25} = 5$ or $x = -5$.
Two solutions : $x = -5$ and $x = 5$.
⚠Solving means finding *all* the solutions.

Answer to Exercise 181 .
$x^2 + 2x = 2x + 1 \iff x^2 = 1 \iff x = \sqrt{1} = 1$ or $x = -1$.
Two solutions : $x = -1$ and $x = 1$.
⚠Solving means finding *all* the solutions.

Answer to Exercise 182 .
$x^2 = 7 \iff x = \sqrt{7}$ or $x = -\sqrt{7}$.
⚠Solving means finding *all* the solutions so do not forget the negative one.
Unless specified otherwise, in Mathematics we want exact values.

Answer to Exercise 183 .
$2x^2 = 50 \iff x^2 = 25 \iff x = \pm\sqrt{25} = \pm 5$
⚠Solving means finding *all* the solutions so do not forget the negative one.

Answer to Exercise 184 .
$2x^2 = 128 \iff x^2 = 64 \iff x = \pm\sqrt{64} = \pm 8$
Two solutions : $x = -8$ and $x = 8$.

Answer to Exercise 185 .
Multiply both sides by 3 :
$\frac{x^2}{3} = 27 \iff x^2 = 81 \iff x = \pm\sqrt{81} = \pm 9$
Two solutions : $x = -9$ and $x = 9$.

Answer to Exercise 186 .
$6x(x+3) = 2x(3x-9) - 45$
$\Leftrightarrow 6x^2 + 18x = 6x^2 - 18x - 45 \Leftrightarrow 36x = -45 \Leftrightarrow x = -\frac{5}{4}$.

Answer to Exercise 187 .

a) $5x^2 - 320 = 0 \Leftrightarrow x^2 = 64$
Two solutions : $x = -8$ and $x = 8$.
⚠Solving means finding *all* the solutions so do not forget the negative one.

b) $5x^2 + 320 = 0 \Leftrightarrow x^2 = -64$
No solution since a square cannot be negative.

c) $5x^3 - 320 = 0 \Leftrightarrow x^3 = 64 \Leftrightarrow x = \sqrt[3]{64} = 3$ (only one solution).

d) $5x^3 + 320 = 0 \Leftrightarrow x^3 = -64 \Leftrightarrow x = \sqrt[3]{-64} = -3$ (only one solution).

Answer to Exercise 188 .

a) $3x(7x-1) - 6(7x-1) = (7x-1)(3x-6)$.

b) $3x(7x-1) - 6(7x-1) = 0 \Leftrightarrow (7x-1)(3x-6) = 0 \Leftrightarrow 7x-1 = 0$ or $3x-6 = 0 \Leftrightarrow x = \frac{1}{7}$ or $x = 2$. (We use $A \times B = 0$ if and only if $A = 0$ or $B = 0$.)

Answer to Exercise 189 .

a) $2x^2 - 13x - 7 = (x-7)(2x+1)$ using PSF method (Product / Sum /Factors) with $P = Product = -14$ and $S = Sum = -13$. The "magic numbers" (the factors) are -14 and 1.

b) $= (x-7)(2x+1) = 0 \Leftrightarrow (x-7)(2x+1) = 0 \Leftrightarrow x = 7$ or $x = -\frac{1}{2}$. (We use $A \times B = 0$ if and only if $A = 0$ or $B = 0$.)

Answer to Exercise 190 .

a) $x - 5$ is a common factor. $x^2 - 25 - 4x(x-5) = (x-5)(x+5) - 4x(x-5) = (x-5)(-3x+5)$

b) $x^2 - 25 - 4x(x-5) = 0 \Leftrightarrow (x-5)(-3x+5) = 0 \Leftrightarrow x = 5$ or $x = \frac{5}{3}$. (We use $A \times B = 0$ if and only if $A = 0$ or $B = 0$.)

Answer to Exercise 191 .

a) $x + 3$ is a common factor. $x^2 - 9 - 4x(x+3) = (x-3)(x+3) - 4x(x+3) = -3(x+1)(x+3)$

b) $x^2 - 9 - 4x(x+3) = 0 \Leftrightarrow -3(x+1)(x+3) = 0 \Leftrightarrow (x+1)(x+3) = 0 \Leftrightarrow x = -3$ or $x = -1$. (We use $A \times B = 0$ if and only if $A = 0$ or $B = 0$.)

Answer to Exercise 192 .
$3x^3 + 5x^2 - 2x = 0$
$\iff x\left(3x^2 + 5x - 2\right) = 0$
$\iff x(x+2)(3x-1)$ [Factorise the bracket by PSF]
$\iff x = 0$ or $x + 2 = 2$ or $3x - 1 = 0$
Three solutions : $x = 0, x = -2$ and $x = \frac{1}{3}$

Answer to Exercise 193 .

a) Method 1 :
$(x+2)^2 = 121 \Leftrightarrow x + 2 = 11$ or $x + 2 = -11$
[If the square of a number is 121 then the number must have been 11 or -11.]
$\Leftrightarrow x = -2 + 11$ or $x = -2 - 11 \Leftrightarrow x = 9$ or $x = -13$.

Method 2 : $(x+2)^2 = 121 \Leftrightarrow (x+2)^2 - 121 = 0$
$\Leftrightarrow (x+2)^2 - 11^2 = 0$ [Difference of squares]
$\Leftrightarrow (x+2+11)(x+2-11) = 0$
$\Leftrightarrow x+13 = 0$ or $x-9 = 0$
$\Leftrightarrow x = -13$ or $x = 9$
In both cases, we get two solutions : $x = -13$ and $x = 9$.

b) $(3x-7)^2 = 81$
$\Leftrightarrow 3x-7 = 9$ or $3x-7 = -9$
$\Leftrightarrow 3x = 16$ or $3x = -2$
$\Leftrightarrow x = \frac{16}{3}$ or $x = -\frac{2}{3}$
We could also do two methods, see above.

Answer to Exercise 194 .

a) Method 1 :
$(x+3)^2 = 49$
$\Leftrightarrow x+3 = 7$ or $x+3 = -7$
[If the square of a number is 49 then the number must have been $+7$ or -7.]
$\Leftrightarrow x = -3+7$ or $x = -3-7$
$\Leftrightarrow x = 4$ or $x = -10$

Method 2 :
$(x+3)^2 = 49$
$\Leftrightarrow (x+3)^2 - 49 = 0$ [Difference of squares]
$\Leftrightarrow (x+3+7)(x+3-7) = 0$
$\Leftrightarrow (x+10)(x-4) = 0$
$\Leftrightarrow x+10 = 0$ or $x-4 = 0$
$\Leftrightarrow x = -10$ or $x = 4$
In both cases, we get two solutions : $x = -10$ and $x = 4$.

b) $(2x+1)^2 = 25$
$\Leftrightarrow 2x+1 = 5$ or $2x+1 = -5$
$\Leftrightarrow 2x = 4$ or $2x = -6$
$\Leftrightarrow x = 2$ or $x = -3$
We could also do two methods, see above.

Answer to Exercise 195 .

a) $x^2 + 6x + 9 = (x+3)^2$.
b) $x^2 - 8x + 16 = (x-4)^2$.
c) $x^2 - 5x + \frac{25}{4} = (x - \frac{5}{2})^2$.

Answer to Exercise 196 .

a) $x^2 + 10x = (x+5)^2 - 25$.
b) $x^2 - 12x = (x-6)^2 - 36$.

Answer to Exercise 197 .

a) $x^2 + 16x = x^2 + 16x + 64 - 64 = (x+8)^2 - 64$.
b) $x^2 - 9x = x^2 - 9x + 4.5^2 - 4.5^2 = (x-4.5)^2 - 20.25$.

Answer to Exercise 198 .

$x^2 + 14x + 1 = 0 \Leftrightarrow (x+7)^2 - 49 + 1 = 0 \Leftrightarrow (x+7)^2 - 48 = 0 \Leftrightarrow (x+7)^2 = 48 \Leftrightarrow x+7 = \sqrt{48}$ or $x+7 = -\sqrt{48}$
$\Leftrightarrow x = -7+\sqrt{48}$ or $x = -7-\sqrt{48}$
$\Leftrightarrow x = -7+4\sqrt{3}$ or $x = -7-4\sqrt{3}$

Answer to Exercise 199 .

- Method 1 : Factorise as a difference of squares and then use $A \times B = 0$ if and only if $A = 0$ or $B = 0$.
- Expand and use quadratic formula. $(2x+1)^2 - (3x-7)^2 = 46x - 5x^2 - 48$.

In both cases, we get two solutions : $x = \dfrac{6}{5}$ and $x = 8$.

Answer to Exercise 200 .

a) $-8x^2 - 80x = -8x(x+10)$.
b) $8(-x+2)(x+12) - 192 = -8x^2 - 80x$.
c) $8(-x+2)(x+12) - 192 = 0 \iff -8x^2 - 80x = 0 \iff -8x(x+10) = 0 \iff x = 0$ or $x = -10$.

Answer to Exercise 201 .

a) $t^2 + 10t + 5 = 0 \iff t = -5 - 2\sqrt{5}$ or $-5 + 2\sqrt{5}$.
b) $-4t^2 - 40t - 20 = 0 \iff t^2 + 10t + 5 = 0$ (just divide both sides by -4) so the solutions are the same : $t \approx -0.53$ or $t \approx -9.47$.

Answer to Exercise 202 .

The solutions of this equation are $x = 0$, $x = \frac{1}{3}$, $x = -2$

Answer to Exercise 203 .

$6x^2 = 36x - 42 \iff 6x^2 - 36x + 42 = 0$
$\iff 6\left(x^2 - 6x + 7\right) = 0$
$\iff x^2 - 6x + 7 = 0$. [Divide by 6]
Using the quadratic equation, the solutions are
$x_1 = \frac{-(-6)+\sqrt{(-6)^2-4\times1\times7}}{2\times1} = 3+\sqrt{2}$
and $x_2 = \frac{-(-6)-\sqrt{(-6)^2-4\times1\times7}}{2\times1} = 3-\sqrt{2}$

Answer to Exercise 204 .

• Method 1 : Completing the square.
$x^2 + 2x + 5 = 0 \Leftrightarrow (x+1)^2 - 1 + 5 = 0 \Leftrightarrow (x+1)^2 + 4 = 0 \Leftrightarrow (x+1)^2 = -40$ which is impossible because the square of real number cannot be negative. Therefore there are no solutions.
• Method 2 : Quadratic formula.
The solutions are $x = \frac{-2\pm\sqrt{2^2-4\times1\times5}}{2\times1}$...but the number inside the square root is negative. That means that there are no solutions.

Answer to Exercise 205 .

a) $t^2 - 6t + 3 = 0 \iff t = 3+\sqrt{6}$ or $t = 3-\sqrt{6}$. (Since $\sqrt{24} = 2\sqrt{6}$, $\frac{6\pm\sqrt{24}}{2} = \frac{6\pm2\sqrt{6}}{2} = \frac{2(3\pm\sqrt{6})}{2} = 3\pm\sqrt{6}$.
b) $-7t^2 + 42t - 21 = 0 \iff t^2 - 6t + 3 = 0$ (just divide both sides by -7) so the solutions are the same : $t = 3+\sqrt{6}$ or $t = 3-\sqrt{6}$.

Answer to Exercise 206 .

a) $x = \frac{2-\sqrt{13}}{3}$ or $x = \frac{2+\sqrt{13}}{3}$

b) No solution. Prove it with the quadratic formula (you get a negative number under the square root) or by completing the square (ideally do both methods so you get some practice.)

c) $\frac{2}{x} - 3 + x = 0 \iff x = 1$ or $x = 2$ (First multiply both sides by x and you get a "normal" quadratic equation.)

d) $x^4 - 3x^2 - 108 = 0 \iff x = -2\sqrt{3}$ or $x = 2\sqrt{3}$ (First, let $u = x^2$ to get a "normal" quadratic equation with variable u and then find x from u.)

Answer to Exercise 207 .

$40x^2 - 40x = -10 \Leftrightarrow 4x^2 - 4x + 1 = 0 \Leftrightarrow (2x-1)^2 = 0 \Leftrightarrow x = \frac{1}{2}$.

Answer to Exercise 208 .

a) $x^2 - 4x + \mathbf{4} = (x-2)^2$

b) $x^2 + 18x + \mathbf{81} = (x+9)^2$

c) $y^2 + 3y + \mathbf{\frac{9}{4}} = \left(y + \frac{3}{2}\right)^2$

d) $y^2 - 5y + \mathbf{\frac{25}{4}} = \left(y - \frac{5}{2}\right)^2$

Answer to Exercise 209 .

$x = 1 + \sqrt{3}$ or $x = 1 - \sqrt{3}$

Answer to Exercise 210 .

By the converse of Pythagoras's theorem, the triangle is right angled if and only if $x^2 + (x+1)^2 = (x+2)^2$.
Let's solve this equation :
$x^2 + (x+1)^2 = (x+2)^2 \Leftrightarrow x^2 + x^2 + 2x + 1 = x^2 + 4x + 4 \Leftrightarrow x^2 - 2x - 3 = 0 \Leftrightarrow (x+1)(x-3) = 0 \Leftrightarrow x = -1$ or $x = 3$.
x is a side of a triangle so $x > 0$, hence there is only one solution, which is $x = 3$.
We recognise the famous Pythagorean triad : 3, 4, 5.

Answer to Exercise 211 .

a) $x = \frac{5}{5+x}$ [B]

b) $x = \frac{5}{5+x}$. Multiply both sides by $(5+x)$ to get $x(5+x) = 5$. i.e. $x^2 + 5x - 5 = 0$. This a quadratic equation. The solutions are $x = \frac{-5+3\sqrt{5}}{2}$ and $x = \frac{-5-3\sqrt{5}}{2}$.
Since $x > 0$ (it is the square root of a number), $x = \frac{5}{5+\frac{5}{5+\frac{5}{5+\dots}}} = \frac{-5+3\sqrt{5}}{2}$

Answer to Exercise 212 .

$x = \sqrt{1+x}$. Squaring both sides, we get $x^2 = 1 + x$
We solve this equation to get : $x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$.
However, $x > 0$ so the only solution is $x = \frac{1+\sqrt{5}}{2}$ (Decimal : $x = 1.61803\dots$)

Answer to Exercise 213 .

a) $X^2 - 13X + 36 = 0 \Leftrightarrow X = 4$ or $X = 9$

b) We want to solve $x^4 - 13x^2 + 36 = 0$. Let $X = x^2$. The equation becomes $X^2 - 13X + 36 = 0$. We know that the solutions are $X = 4$ or $X = 9$, i.e. $x^2 = 4$ or $x^2 = 9$. Hence there are four solutions : $x = -2, x = 2, x = -3, x = 3$.

c) We want to solve $\frac{1}{t^2} - 13 \times \frac{1}{t} + 36 = 0$. Let $X = \frac{1}{t}$. The equation becomes $X^2 - 13X + 36 = 0$. We know that the solutions are $X = 4$ or $X = 9$, i.e. $\frac{1}{t} = 4$ or $\frac{1}{t} = 9$. Hence there are two solutions : $t = \frac{1}{4}, t = \frac{1}{9}$.

Answer to Exercise 214 .

Multiply both sides of the equation by $x^2 - 9$ to get rid of the denominators :
$\frac{x}{x^2-9} + \frac{1}{x+3} = 1 \iff 2x - 3 = x^2 - 9 \iff x^2 - 2x - 6 = 0$.
The solutions of this quadratic equation are $x = 1 + \sqrt{7}$ and $x = 1 - \sqrt{7}$

Answer to Exercise 215 .

$\frac{7x}{x-1} - \frac{2x+1}{x+1} = 2$
$\Leftrightarrow 7x(x+1) - (2x+1)(x-1) = 2(x-1)(x+1)$ [Multiply both sides by $(x+1)(x-1)$]
$\Leftrightarrow 5x^2 + 8x + 1 = 2x^2 - 2 \Leftrightarrow 3x^2 + 8x + 3 = 0$
Two solutions : $x = \frac{-4+\sqrt{7}}{3}$, $x = \frac{-4-\sqrt{7}}{3}$

Answer to Exercise 216 .

$\frac{7x}{x+2} - \frac{2x+1}{x-2} = 2$
[Multiply both sides by $(x+2)(x-2)$ and then simplify to get a quadratic equation]
$3x^2 - 19x + 6 = 0$
Two solutions : $x = 6$, $x = \frac{1}{3}$

Answer to Exercise 217 .

$\frac{u+3}{2u-7} = \frac{2u-1}{u-3} \Leftrightarrow (u+3)(u-3) = (2u-7)(2u-1)$
$\Leftrightarrow u^2 - 9 - 4u^2 + 16u - 7 = 0 \Leftrightarrow -3u^2 + 16u - 16 = 0$
$\Leftrightarrow u = 3$ or $u = \frac{7}{2}$

Answer to Exercise 218 .

a) Solve Multiply both sides by $6(a+3)$ to get :
$12 + 3(a+3)^2 = 20(a+3)$
Two solutions : $a = 3$, $a = -\frac{7}{3}$

b) Let $u = a^2$. By part (1.), $u = 3$ or $u = -\frac{7}{3}$.
$\therefore a^2 = 3$ or $a^2 = -\frac{7}{3}$.
A square cannot be negative, so the equation $a^2 = -\frac{7}{3}$ has NO solution.
Hence, there are only two solutions :
$a = \sqrt{3}$, $a = -\sqrt{3}$

Answer to Exercise 219 .

$\frac{\frac{1}{x} - \frac{2}{3}}{\frac{1}{x} + \frac{2}{3}} = \frac{x-2}{x+2}$
First, multiply the numerator and the denominator of the fraction on the left hand side by $3x$, to get :
$\frac{3-2x}{3+2x} = \frac{x-2}{x+2}$
Then cross multiply, simplify, to get : $x^2 = 3$
Two solutions : $x = \sqrt{3}$, $x = -\sqrt{3}$

Answer to Exercise 220 .

$\frac{\frac{1}{x}-\frac{2}{3}}{\frac{1}{x}+\frac{2}{3}}=\frac{x+2}{x-+2}$

First, multiply the numerator and the denominator of the fraction on the left hand side by $3x$, to get :

$\frac{3-2x}{3+2x}=\frac{x+2}{x-2}$

Then cross multiply, simplify, to get : $x^2=-3$

A square cannot be negative so no solutions.

Answer to Exercise 221 .

Let s = unknown side length of square tile.

$200s^2=128(s+1)^2$

$9s^2-32s-16=0$

$(9s^2+4)(s-4)=0$

Two solutions; $s=-4/9$ and $s=4$.

Since we can't have a negative size of a physical object, we'll throw that one out. So $s=4$.

Each tile was 4 cm by 4 cm before it gets increased.

Answer to Exercise 222 .

Dividing by zero is not allowed, so the values $x=-11$ and $x=11$ (solutions of $x^2-121=0$) are not allowable as substitutions for the variable x.

Answer to Exercise 223 .

Dividing by zero is not allowed, so the values $x=-5$ and $x=5$ (solutions of $50-2x^2=0$) are not allowable as substitutions for the variable x.

Answer to Exercise 224 .

$\frac{1}{q}+10=0 \iff 1+10q=0$ (multiply both sides by q).

The solution is $q=-\frac{1}{10}$.

Dividing by zero is not allowed, so the value $q=-1/10$ (solution of $\frac{1}{q}+10=0$) is not allowable as substitutions for the variable q.

Answer to Exercise 225 .

Dividing by zero is not allowed, so the values $x=-8$ and $x=8$ (i.e. the solutions of $x^2-64=0$) must be excluded from the domain.

Domain = All real numbers except -8 and 8.

This can also be written :

Domain : $x\in\mathbb{R}, x\neq 8, x\neq -8$

Answer to Exercise 226 .

Dividing by zero is not allowed, so the the solutions of $x^2+-64=0$) must be excluded from the domain. Since this equation has no solution, no number needs to be excluded.

Domain = All real numbers.

This can also be written :

Domain : $x\in\mathbb{R}$.

Answer to Exercise 227 .

Expand and equate the coefficients, you get $a=1$, $b=-6$ and $c=9$ so $n^2\equiv(n+3)^2-6(n+3)+9$

Answer to Exercise 228 .

Sketch the graph of $y=(x-3)(x+5)$ to get :

$x<5$ or $x>3$, which can be rewritten :

$x\in(-\infty,-5)\bigcup(3,\infty)$.

Answer to Exercise 229 .

Sketch the graph of $y=x^2+5x+4=(x+1)(x+4)$ to get :

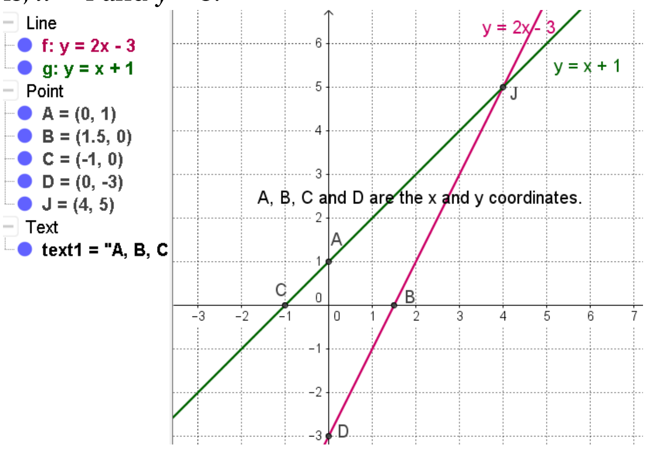
a) $x^2+5x+4\leqslant 0 \Leftrightarrow x\in[-4,-1]$

b) Multiply $-x^2-5x-4\geqslant 0$ by -1 to see that it is equivalent to the previous one so the solutions are the same.

c) Multiply $-x^2-5x-4<0$ by -1. This yields $x^2+5x+4>0$ and the solutions can now be read on the graph $y=x^2+5x+4=(x+1)(x+4)$. The set of solutions is $x\in(-\infty,-4)\bigcup(-1,\infty)$.

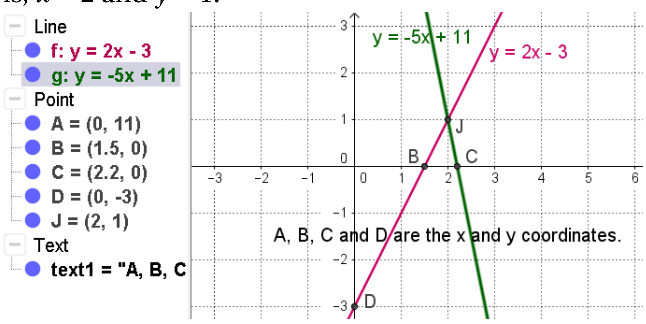
Answer to Exercise 230 .

The coordinates of the point of intersection of the straight lines are the solutions of $\begin{cases} y=2x-3 \\ y=x+1 \end{cases}$ that is, $x=4$ and $y=5$.



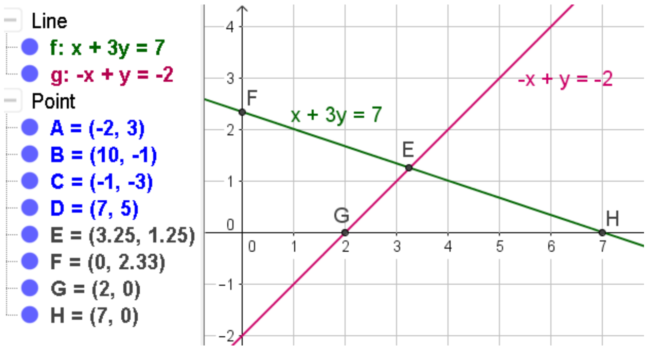
Answer to Exercise 231 .

The coordinates of the point of intersection of the straight lines are the solutions of $\begin{cases} y=2x-3 \\ y=-5x+11 \end{cases}$ that is, $x=2$ and $y=1$.



Answer to Exercise 232 .

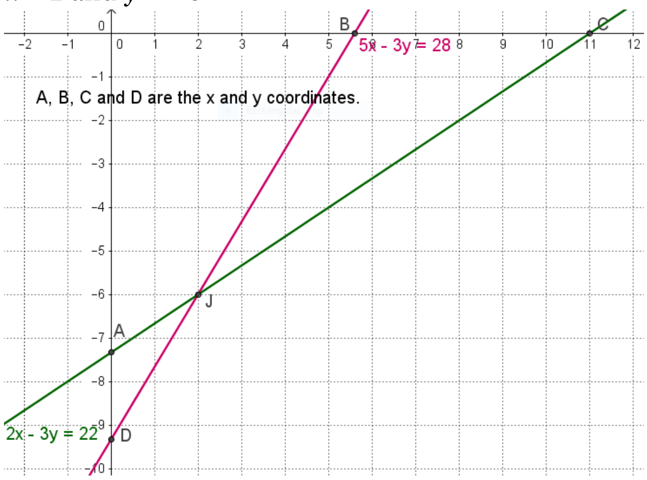
The coordinates of the point of intersection of the straight lines are the solutions of $\begin{cases} x+3y=7 \\ -x+y=-2 \end{cases}$ that is, $x=3.25=3\frac{1}{4}=\frac{13}{4}$ and $y=1.25=1\frac{1}{4}=\frac{5}{4}$.



Answer to Exercise 233 .
$x = 5$ and $y = \frac{18}{5}$

Answer to Exercise 234 .
$x = 7$ and $y = 5$

Answer to Exercise 235 .
$x = 2$ and $y = -6$



Answer to Exercise 236 .
$x = 1$ and $y = 6$

Answer to Exercise 237 .
$72 min = 1\ h\ 12$

Answer to Exercise 238 .
$x = \frac{122}{99}$

Answer to Exercise 239 .
$x = x = 78.8\dot{3} = 78.833333333....$

Answer to Exercise 240 .
$\sqrt{48} + 2\sqrt{75} = 4\sqrt{3} + 2 \times 5\sqrt{3} = 14\sqrt{3}.$

Answer to Exercise 241 .
$\frac{\sqrt{22}}{\sqrt{176}} = \sqrt{\frac{22}{176}} = \sqrt{\frac{1}{8}} = \frac{\sqrt{1}}{\sqrt{8}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}} = \frac{1\times\sqrt{2}}{2\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{2}}{4}.$

Answer to Exercise 242 .

Answer to Exercise 243 .

Answer to Exercise 244 .

Answer to Exercise 245 .

Answer to Exercise 246 .

Answer to Exercise 247 .

Answer to Exercise 248 .

Answer to Exercise 249 .

Answer to Exercise 250 .

Answer to Exercise 251 .

a) $3x^4 \times 5x^6 = 3 \times 5 \times x^4 \times x^6 = 15x^{4+6} = 15x^{10}$
b) $3x^4 + 5x^6$ cannot ne simplified (they are NOT like terms).
c) $3x^0 + (5x)^0 = 3 \times 1 + 1 = 3 + 1 = 4$
d) $3^4 \times 3^6 = 3^{4+6} = 3^{10}$
e) $(5^4)^2 = 5^{4\times 2} = 5^8$
f) $(3^4)^8 \times 3^6 = 3^{32} \times 3^6 = 3^{32+6} = 3^{38}$
g) $(3^2)^4 \times 9^5 = 3^{4\times 2} \times (3^2)^5 = 3^8 \times 3^{10} = 3^{18}$

Answer to Exercise 252 .

Answer to Exercise 253 .

Answer to Exercise 254 .
$A = \frac{3^8 \times 3^2}{3^7} = 3^3 = 27 > 25 = 5^2 = B$

Answer to Exercise 255 .

a) $5^{-9} \times 5^6 = 5^{-3} = \frac{1}{5^3}$
b) $2x^7 \times -5x^{-11} \times 3x^0 = -30x^{-4} = -\frac{30}{x^4}$
c) $\frac{4^{17}}{4^{-12}} = 4^{29}$
d) $\frac{4^2 + 1^{-3}}{4^0} = 17$
e) $7x^{-3} \times 3x^{12} = 21x^9$
f) $7x^{-2} + 3x^{-2} = 10x^{-2} = \frac{10}{x^2}$
g) $\frac{(3^{-8} \times 3^{-4})^0}{3^{-5}} = \frac{1}{3^{-5}} = 3^5$
h) $\frac{3^2 \times (3^4)^0}{3^{-5}} = 3^7$
i) $13^4 \times 13^{-8} = 13^{-4} = \frac{1}{13^4}$
j) $(2^4)^{-6} = 2^{-24}$
k) $(4^2)^{-7} = 4^{-14} = (2^2)^{-14} = 2^{-28}$
l) $\left(\frac{5^{-7}}{2}\right)^3 = \frac{5^{-21}}{2^3}$
m) TRUE : $5^{-3} \times 2^{-3} = \frac{1}{5^3 \times 2^3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$

Answer to Exercise 256 .

a) $25^{\frac{1}{2}} = 5$
b) $8^{\frac{1}{3}} = 2$
c) $8^{\frac{2}{3}} = 2^2 = 4$
d) $1000^{\frac{2}{3}} = 10^2 = 100$

e) $\left(81x^{12}\right)^{\frac{1}{2}} = 9x^6$

f) $\left(121x^4y^6\right)^{\frac{1}{2}} = 121^{\frac{1}{2}}\left(x^4\right)^{\frac{1}{2}}\left(y^6\right)^{\frac{1}{2}} = 11x^2y^3$

g) $\sqrt{49t^8} = \left(49t^8\right)^{\frac{1}{2}} = 49^{\frac{1}{2}}\left(t^8\right)^{\frac{1}{2}} = 7t^4$

h) $\sqrt{x} \times x^{\frac{7}{2}} = x^{\frac{1}{2}} \times x^{\frac{7}{2}} = x^{\frac{8}{2}} = x^4$

i) $\sqrt[3]{x} \times x^{\frac{5}{3}} = x^{\frac{1}{3}} \times x^{\frac{5}{3}} = x^{\frac{6}{3}} = x^2$

j) $\sqrt{6x} \times (6x)^{\frac{3}{2}} = 6^{\frac{1}{2}}x^{\frac{1}{2}}6^{\frac{3}{2}}x^{\frac{3}{2}} = 6^{\frac{4}{2}}x^{\frac{4}{2}} = 36x^2$

Answer to Exercise 257 .

a) $A = B = 1$ according to calculator.

b) i) Let $x = 10^{-8}$. $A = (1-2x)(1+2x) = 1-4x^2$ and $B = A^2 = (1-4x^2)^2 = 1-8x^2+16x^4$.

ii) Exact values of $A = 1-4x^2 = 1-4\times 10^{-16}$ and B :

iii) First, note that $A \neq B$. Also, $0 < A < 1$ so $A^2 < A$. (Multiply the inequality by the positive number A). Therefore, $A > B$.