Solving Quadratic Equations

There are three methods to solve a ***quadratic*** *equation, that is an equation of the form* $ax^2 + bx + c = 0$.

■ Method 1: ***Factorising***

(1) Move everything to the Left Hand Side (LHS), to make your equation look like $ax^2 + bx + c = \mathbf{0}$

(2) Factorise the LHS using PSF if possible (*if factorising is not possible with PSF, use the quadratic formula, see below*)

(3) Use the fact that if a product is equal to zero, at least one of its factors must be equal to zero. (*This is true only for zero: you could have a product equal to, say, 3 without any of the factors being equal to 3 :* $6 \times \frac{1}{2} = 3;\ -3\ \times -1 = 3, \frac{3}{10} \times 10 = 3, .. etc.$).

■ Method 2: ***Completing the square*** (*This uses the perfect square formula* $(x + d)^2 = x^2 + 2dx + d^2$).

(1) Move the constant term to Right Hand Side (RHS).

(2) Halve the coefficient of x, this gives you the number d in $x^2 + 2dx + d^2$

(3) Add d^2 to both sides, the LHS is now a perfect square as LHS = $x^2 + 2dx + d^2 = (x + d)^2$ and the equation has become $(x + d)^2 = p$.

(4) If $p \geq 0$, we get $x + d = \pm\sqrt{p}$, so the solutions are $x = -d \pm \sqrt{p}$ (*and if* $p < 0$, *then the equation has no solution*.)

■ Method 3: ***Quadratic Formula***.

(1) Move everything to the LHS, to make your equation look like $ax^2 + bx + c = \mathbf{0}$

(2) The solutions of the equation $ax^2 + bx + c = 0$ are $\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

Note: If the number under the square root is positive then the equation has two solutions. If the number under the square root is zero, then the equation has exactly one solution. If the number under the square root is negative, then the equation has no solution.