Summary	Consumer Arithmetic Simple interest and buying on Terms Compound interest and Depreciation	Y10

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I. PERCENTAGES

INCREASING OR DECREASING A QUANTITY BY SOME PERCENTAGE

Example: To increase a quantity by 12%, you multiply it by $1 + 12\% = 1 + 0.12 = 1.12$

To decrease a quantity by 12%, you multiply it by $1 - 12\%\ 1 - 0.12 = 0.88$.

General case

To increase a quantity by r %, you multiply it by $1 + r$, where r is expressed as a decimal.
To decrease a quantity by r %, you multiply it by $1 - r$, where r is expressed as a decimal.

○ Example 1.

To increase a quantity by 25%, you multiply it by _______

To decrease a quantity by 25%, you multiply it by _______

○ Example 2.

An item cost \$56. If its price increases by 15%, the new price will be \$ ______________ = \$ ______.

An item cost \$56. If its price is reduced by 15%, the new price will be \$ ____________ = \$ _______.

Note that it makes sense: If the price is reduced by 15%, it means you get 15% for free but you still need to pay for the remaining 85%.

II. SIMPLE INTEREST AND COMPOUND INTEREST

- When you invest money, you **receive interest** from your investment.
- When you borrow money, you **pay interest** on your loan.
- The original amount of money invested or borrowed is called the principal.
- This interest rate is a percentage of the principal, usually written as a rate per annum ('per year'), abbreviated 'p.a.'

Simple interest is calculated on the *original* principal only whereas ***compound interest*** is calculated on the on the original principal *as well as on any accumulated interest*.

○ Example 3.

► **Simple interest**: \$1000 is invested at 3% p.a. *simple* interest (*p.a.* means "per annum" = "per year").
The table below shows the amount on the bank account.

Principal	+\$30	*After 1 year*	+\$30	*After 2 years*
\$1000	Interest = $3\%\ of\ \$1000 = \$30$	\$1030	Interest will always be 3% of INITIAL amount	\$1060

After 3 years, the amount will be \$ ____________________ = \$ _________.

After 4 years, the amount will be \$ ____________________ = \$ _________.

After N years, the amount will be \$ ___________________.

► **Compound interest**: $1000 is invested at 3% p.a. with interest *compounded* annually.

Principal	× 1.03	*After 1 year*	× 1.03	*After 2 years*
$1000	Interest = $3\%\ of\ \$1000 = \$30$	$1030	Interest will always be 3% of previous amount WHICH INCLUDES INTERESTS, in our case, on $1030	$1060.9

After 3 years, the amount will be $ ____________________ = $ _________.

After 4 years, the amount will be $ ____________________ = $ _________.

After N years, the amount will be $ ______________.

■ The examples above show you that you can rely on common sense. If you prefer to have a formula:

SIMPLE INTEREST FORMULA

$I = PRN$, where:
- I is the simple interest,
- P is the principal,
- R is the interest rate per time period, expressed as a fraction or decimal, and
- N is the number of time periods

COMPOUND INTEREST FORMULA

$A = P(1 + R)^n$, where:
- A is the total (final) amount of the investment
- P is the principal
- R is the interest rate per compounding period, expressed as a decimal
- n is the number of compounding periods

The compound interest is then calculated using this formula:

$$\text{Compound interest} = \text{total amount} - \text{principal}$$
$$I = A - P$$

○ *Example 4. What if the interest is not paid annually but monthly?*

$1000 is invested for 3 years at 6% p.a. interest compounded monthly. How much money will you have in the end? How much interest have you earned over 3 years?

Total amount of the investment: $______________

Total interest earned: $______________________________ = $_______________

WHAT IF THE INTEREST IS PAID MONTHLY? *R and n must be expressed in months.*
- Find the number of time periods: Multiply the number of years by 12 to get the *number of months*.
- Find the *monthly* interest rate: Just divide the annual interest rate by 12.

Note: With the same annual interest rate, more interest will be earned if the interest is compounded monthly than if the interest is compounded annually.

III. BUYING ON TERMS WORKS LIKE SIMPLE INTEREST

Many customers buy expensive household items **on terms**, which means 'paying off' the item by regular instalments over time, after paying a deposit. A **term payments** plan is also called **hire purchase** because the purchaser actually hires the item until it is completely paid off. Special offers can include interest-free periods, but there may be other conditions such as establishment fees and extra charges if the regular repayments are not paid on time. Also, if the purchaser fails to keep up with the payments, higher interest may be charged or the item may be **repossessed** (taken back).

Instalments or *repayments* are the amount of money paid at regular time intervals to pay off a loan.

With a **deferred payment plan**, the customer does not make any repayments until a later date, such as after three years.

Deferred means 'delayed'.

Buying on Terms works like simple interest: If you borrow P dollars for N years at a flat rate of R p.a., the interest you will have to pay is $I = PRN$.

IV. DEPRECIATION WORKS LIKE COMPOUND INTEREST

Depreciation is the decrease in value of an item over time. When items we buy lose value because of age or frequency of use, they are said to **depreciate**.
The compound interest formula can be adapted to find the depreciated value of an item. While compound interest involves repeated percentage increases, depreciation involves repeated percentage *decreases*, so its formula has a minus sign.

◯ *Example 5.*: A computer bought \$2000 depreciates at 20% p.a., which means it loses every year 20% of what it was worth the previous year (NOT 20% of what it was worth initially). In other words, each year is only worth 80% of what it was worth one year earlier.

Principal	$\times (1 - 20\%) = \times 0.80$	*After 1 year*	$\times 0.80$	*After 2 years*
\$2000	Depreciation = 20% *of* \$2000 = \$400	\$1600	Depreciation will always be 20% of previous amount WHICH INCLUDES PREVIOUS DEPRECIATION.	\$1280

After 3 years, the computer will be valued \$ ____________________ = \$ _________.

After 4 years, the computer will be valued \$ ____________________ = \$ _________.

The depreciation over 4 years will be \$ ______________.

■ The formula below says that if the original value of an item was P dollars, after n years, with a rate of depreciation of R p.a., it will only be worth $A = P(1 - R)^n$.

DEPRECIATION FORMULA

$A = P(1 - R)^n$, where:

- A is the final value of the item
- P is the original value of the item
- R is the rate of depreciation per period, expressed as a decimal
- n is the number of periods of depreciation

The amount of depreciation is then calculated using this formula:

Depreciation = original value − final value