| Summary | Graphs | Y10 |
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## STRAIGHT LINES

*Recall* : A relation is ***linear*** if and only if its equation can be written $ax + by + c = 0$, where $a$ and $b$ are not both zero.

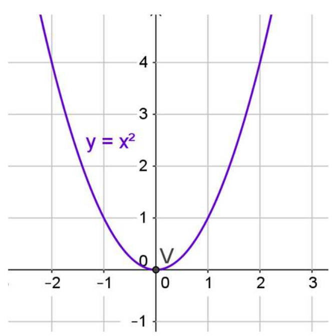
The graph of a linear relation is a straight line.

- If the equation can be written $y = k$, then its graph is a *horizontal* line.
- If the equation can be written $x = k$, then its graph is a *vertical* line.
- If the equation can be written $y = mx + b$, then its graph is a straight line with gradient $m$ and $y$-intercept $b$.

## PARABOLAS

### Parabolas with equation in Vertex Form

The star of the family is $y = x^2$

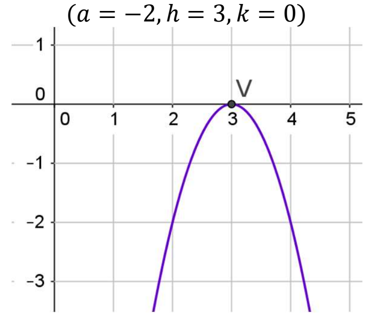



The point V is the ***vertex*** or ***turning point***.

$a = 1 > 0$,
parabola concave up

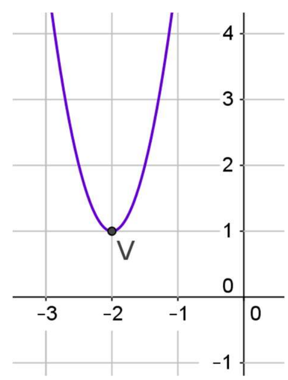
The vertex is $V(0, 0)$

$y = -2(x-3)^2$
$(a = -2, h = 3, k = 0)$




$a = -2 < 0$,
parabola concave down
The vertex is $V(3, 0)$
*Solve bracket =0, ie. x-3=0*

$y = 4(x+2)^2 + 1$
$(a = 4, h = -2, k = 1)$




$a = 4 > 0$,
parabola concave up
The vertex is $V(-2, 1)$
*Solve bracket =0, ie.x+2=0*

If the equation of parabola is of the form $y = a(x-h)^2 + k$ (vertex form), the $x$-coordinate of the vertex is the value of $x$ that makes the bracket zero.

*Let's see why:*

$y = x^2 \geq 0$ because a square must be greater than or equal to zero. Therefore, the lowest point of the parabola (the vertex) is the one for which $y = x^2 = 0$, that is $x = 0$.

$y = -2(x-3)^2 \leq 0$ so the highest point of the parabola (the vertex) is the one for which $y = -2(x-3)^2 = 0$, that is $x - 3 = 0$, that is $x = 3$.

$y = 4(x+2)^2 + 1 \geq 1$ because $4(x+2)^2 \geq 0$. Therefore, the lowest point of the parabola (the vertex) corresponds to $4(x+2)^2 + 1 = 1$, that is $x + 2 = 0$, that is $x = -2$

$y = a(x-h)^2 + k$ is the equation of a ***parabola*** (*in vertex form*).

- If the coefficient of $x^2 = a > 0$, the parabola is ***concave up*** (happy face)
- If the coefficient of $x^2 = a < 0$, the parabola is ***concave down*** (sad face)
- The vertical line $x = h$ is an axis of symmetry for the parabola. It goes through the vertex.
- The vertex has coordinates $(h, k)$ (*h is the value of x that makes the bracket zero*)
- The $x$-intercepts, if any, can be found by factorising using the difference of squares or, of course, using the quadratic formula.

◯ *Example 1. Parabolas with equation in vertex form (i.e. in the form* $y = a(x-h)^2 + k$*)*

**1)** The graph of $y = 6x^2$ is a parabola concave _______________ (*up/down*) with vertex $V$(_____,_____). The equation of its axis of symmetry is __________________.

**2)** The graph of $y = 2(x-1)^2 - 7$ is a parabola concave _______________ (*up/down*) with vertex $V$(_____,_____). The equation of its axis of symmetry is __________________.

**3)** The graph of $y = -2(x-5)^2 + 3$ is a parabola concave _______________ (*up/down*) with vertex $V$(_____,_____). The equation of its axis of symmetry is __________________.

**4)** The graph of $y = 4(x+7)^2 - 9$ is a parabola concave _______________ (*up/down*) with vertex $V$(_____,_____). The equation of its axis of symmetry is __________________.

## Parabolas with Equation in Expanded Form

In $y = ax^2 + bx + c$ (*with* $a \neq 0$, *otherwise, we get a straight line*) the highest power of $x$ is 2 and for this reason, it is called a ***quadratic equation***.

The graph of a quadratic equation is a ***parabola***.
(*In all that follows, a, b and c are the coefficients in* $y = ax^2 + bx + c$)

- If the coefficient of $x^2 = a > 0$, the parabola is concave up (happy face)
- If the coefficient of $x^2 = a < 0$, the parabola is concave down (sad face)
- The vertical line $x = -\frac{b}{2a}$ is an axis of symmetry for the parabola. It goes through the vertex.
- The $x$-coordinate of the vertex is $x_V = -\frac{b}{2a}$ (and you get the $y$-coordinate of the vertex by substituting the value of $x$ into $y = ax^2 + bx + c$.
- The $x$-intercepts, if any, can be found using the quadratic formula.

◯ *Example 2. A parabolas with equation of the form* $y = ax^2 + bx + c$

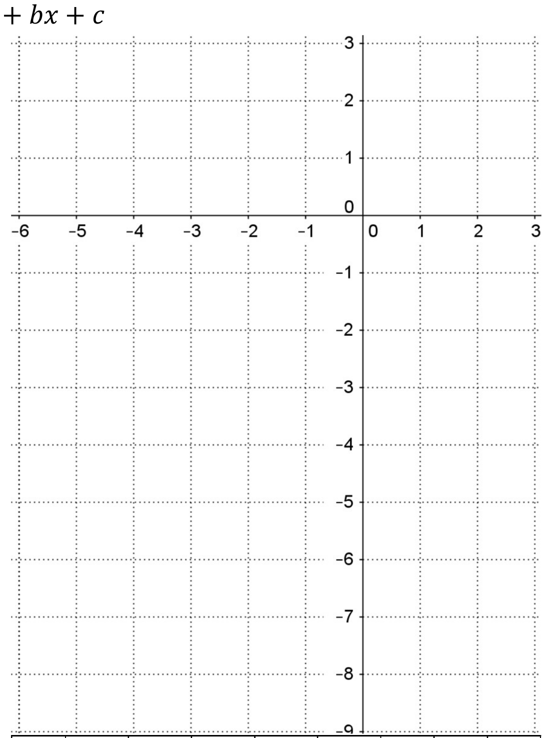
Let $y = x^2 + 4x - 4$

**1)** $y = x^2 + 4x - 4 = ax^2 + bx + c$ with $a =$_______, $b =$______ and $c =$__________.

**2)** What is the equation of the axis of symmetry?

**3)** Find the coordinates of the vertex.

**4)** Find the $x$ and $y$ intercepts if any.




| $x$ | $-5$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | |

where $y = x^2 + 4x - 4$

**5)** Complete the following table of values.

**6)** Sketch the graph.

■ From $y = ax^2 + bx + c$ (***expanded form***) to $y = a(x-h)^2 + k$ (***vertex form***) and vice-versa:

- If you want to rewrite $y = a(x-h)^2 + k$ in the form $y = ax^2 + bx + c$, just expand!
- If you want to rewrite $y = ax^2 + bx + c$ in the form $y = a(x-h)^2 + k$, factorise $a$ and then complete the square (*halve the coefficient of x, square it, add and subtract the result to* $y = ax^2 + bx + c$ *so a perfect square appears*). The vertex form makes it easy to find the vertex.

◯ *Example 3*. Write $y = x^2 + 4x - 4$ in vertex form and use it to find the vertex (this is the graph from the previous example).

**Note: How to use your *fx*-100 AU calculator to fill in a table of values**.

Say you want to fill in a table of values for $y = 2x^2 - 3x + 4$

(1) Enter your equation in the calculator and then press ENTER.

(Eg: For $y = 2x^2 - 3x + 4$, enter: 2, ALPHA , X, , -3, ALPHA, X, +4 then ENTER)

*A random number will appear on your screen, ignore it.*

(2) Enter a value into the variable X in the calculator

(Eg: To enter $X = 5$, do: 5, Shift, STO , X; On screen you see 5→X)

*NB: Don't press the ALPHA key this time when you enter X.*

(3) Use the top arrow of  to go back to your expression (E.g. $2x^2 - 3x + 4$)

(4) Press ENTER, so $2x^2 - 3x + 4$ is evaluated for the value of X you chose.

*(with* $y = 2x^2 - 3x + 4$ *and* $X = 5$ *you should get 39)*

(5) Repeat with a different value of $x$ until your table of values is filled.

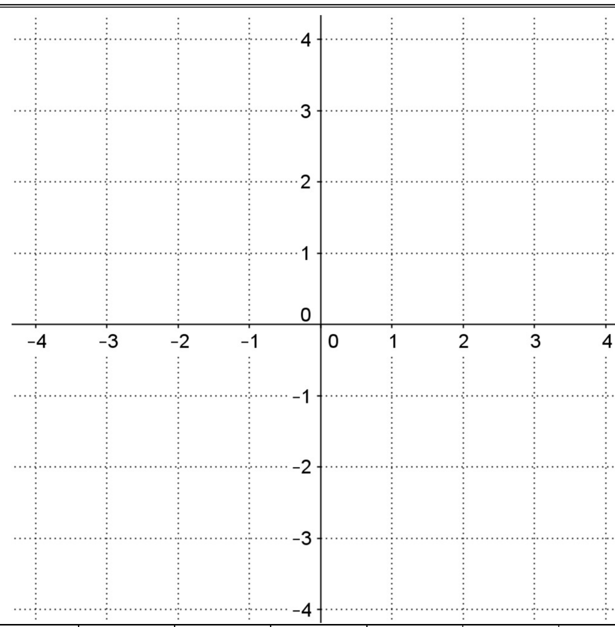
◯ *Example 4*. Use this method to check your table of values from example 2 (with $y = x^2 + 4x - 4$).

## HYPERBOLAS

◯ *Example 5. The most famous hyperbola!*

Let $y = \frac{1}{x}$.

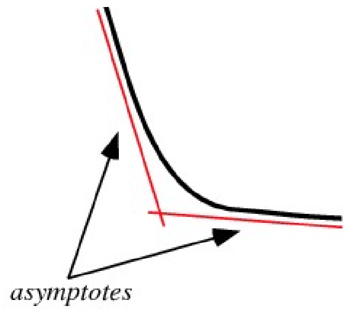
Complete the table of value below and then sketch the graph.

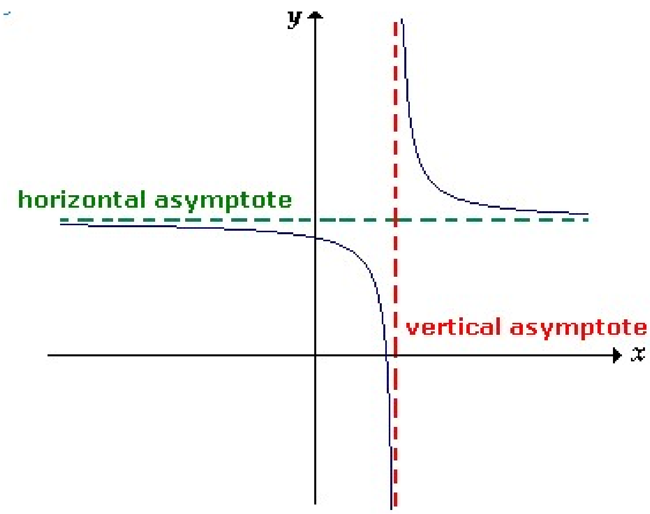


| $x$ | $-3$ | $-2$ | $-1$ | $-\frac{1}{2}$ | $-\frac{1}{4}$ | $0$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y = \frac{1}{x}$ | | | | | | | | | | | |

## Asymptotes

An ***asymptote*** is, essentially, a line that a graph approaches arbitrarily closely.

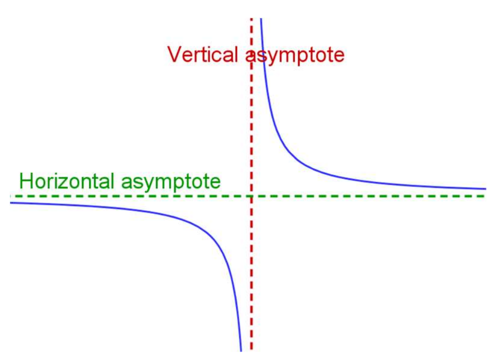


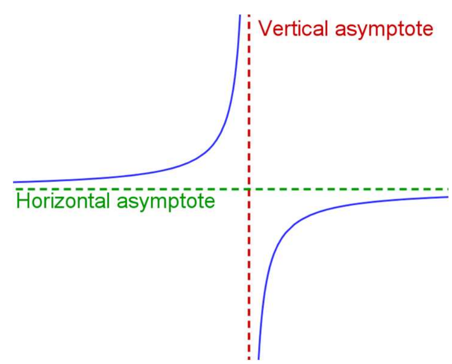




- **Vertical asymptotes** correspond to values that would lead to a division by zero. Vertical asymptotes occur if $y$ approaches $+\infty$ or $-\infty$ when $x$ approaches such a "forbidden" value.
- **Horizontal asymptotes** occur if $y$ approaches a finite value when $x$ approaches $+\infty$ or $-\infty$.

## Hyperbolas

The ***reciprocal*** function $y = \frac{1}{x}$ is the star of this family of functions. A graph with an equation of the form $y = \frac{a}{bx+c} + d$ or $y = \frac{ax+b}{cx+d}$ is a ***hyperbola***. It has a vertical and a horizontal asymptotes and looks like one of these two graphs ("*decreasing on both sides of the vertical asymptote*" for the first one and ("*increasing on both sides of the vertical asymptote*" for the second one):




- Use the "forbidden" value (leading to division by zero) to find the **vertical asymptote**.
- The (finite) value which $y$ approaches when $x$ approaches $+\infty$ or $-\infty$ gives you the **horizontal asymptote.**
- Use the value at a point to decide between the two possible shapes.

Example 6.

**1)** The vertical asymptote of $y = 7 + \frac{1}{x+3}$ is ___________ and its horizontal asymptote is _________.

**2)** The vertical asymptote of $y = -2 + \frac{1}{x-5}$ is __________ and its horizontal asymptote is _________.

**3)** The vertical asymptote of $y = 5 - \frac{1}{2x+4}$ is __________ and its horizontal asymptote is _________.