Algebra Review: How to factorise?

• **Common factor** *(Always try first)*	Eg : $20x - 12x^2 = 5 \times \mathbf{4x} - 3x \times \mathbf{4x} = \mathbf{4x}(5 - 3x)$
• **Grouping in pairs** *(Try if 4 terms)*	Eg : Factorize $7x - 21 + x^2 - 3x$ $= 7(x - 3) + x(x - 3) = (7 + x)(x - 3)$
• **PSF** (Product, Sum Factors) for expressions of the form $ax^2 + bx + c$ *(Try if there are three terms and one of them is an x^2)*	*Idea: Rewrite the coefficient of x as a sum of two numbers and then factorise in pairs. How to find these numbers?* • Their Product = Coefficient of x^2 × *constant term* $(i.e.$ *the one with no* $x)$ • Their Sum = Coefficient of x Eg : Factorize $\mathbf{2}x^2 - \mathbf{7}x - 15$ $\boldsymbol{Product} = \mathbf{2} \times -15 = -30$ $\boldsymbol{Sum} = \mathbf{-7}$ $\boldsymbol{Factors}\ (must\ guess) : -10\ and\ 3$ $2x^2 - \mathbf{7x} - 15 = 2x^2 - \mathbf{10x} + \mathbf{3x} - 15$ $= 2x(x - 5) + 3(x - 5)$ $= (2x + 3)(x - 5)$
• **Difference of squares** $\boldsymbol{A^2 - B^2 = (A + B)(A - B)}$	Eg : $x^2 - 16 = (x + 4)(x - 4)$ Eg : $49x^2 - 25y^2 = (7x + 5y)(7x - 5y)$

Your turn, work in your book :

◯ Exercise 1.

Factorize fully

(i) $56x^2 + 72xy$

(ii) $x^2 - 9x - 22$

(iii) $mn + 3m - 8n - 24$

(iv) $x^2 - 49$

(v) $49 - x^2$

(vi) $9x^2 - 16$

(vii) $x^2 - 5x - 14$

(viii) $4x^2 + 12x + 9$

(ix) $3x^2 - 4x - 15$

(x) $4x^2 - 4x - 15$

(xi) $10x^2y^3 - 250y^3$

◯ Exercise 2.

Factorise fully the following expressions:

(i) $2x^2 + x - 15$

(ii) $x^2 - 9$

Hence simplify

(iii) $\frac{2x^2+x-15}{x^2-9}$

◯ Exercise 3.

Express the following as single fractions:

(i) $\frac{7}{6x} + \frac{3}{x}$

(ii) $\frac{7}{6x^2} - \frac{3}{x^2}$

Hence simplify

(iii) $\left(\frac{7}{6x} + \frac{3}{x}\right) \div \left(\frac{7}{6x^2} - \frac{3}{x^2}\right)$

Examples that shows how to use the **PSF method (Product / Factor / Sum)** [link in DoNow]

http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-factorisingquadratics.pdf

Solutions

◯ Exercise 1.

Factorize fully

(i) $56x^2 + 72xy = 8x(7x + 9y)$ [Common Factor $8x$]

(ii) $x^2 - 9x - 22 = (x - 11)(x + 2)$ [PSF : $P = -22, S = -9, F = -11 \text{ and } 2$]

(iii) $mn + 3m - 8n - 24 = (m - 8)(n + 3)$ [Grouping in pairs]

(iv) $x^2 - 49 = (x - 7)(x + 7)$ [Difference of Squares]

(v) $49 - x^2 = (7 - x)(7 + x)$ [Difference of Squares]

(vi) $9x^2 - 16 = (3x - 4)(3x + 4)$ [Difference of Squares]

(vii) $x^2 - 5x - 14 = (x + 2)(x - 7)$ [PSF : $P = -14, S = -5, F = -7 \text{ and } 2$]

(viii) $4x^2 + 12x + 9 = (2x + 3)^2$ [Perfect Square or PSF]

(ix) $3x^2 - 4x - 15 = (x - 3)(3x + 5)$ [PSF : $P = -45, S = -4, F = -9 \text{ and } 5$]

(x) $4x^2 - 4x - 15 = (2x - 5)(2x + 3)$ [PSF : $P = -60, S = -4, F = -10 \text{ and } 6$]

(xi) $10x^2y^3 - 250y^3 = 10y^3(x^2 - 25) = 10y^3(x - 5)(x + 5)$
[Common Factor $10y^3$ then Difference of Squares]

◯ Exercise 2.

Factorise fully the following expressions:

(i) $2x^2 + x - 15 = (x + 3)(2x - 5)$ [PSF : $P = -30, S = 1, F = 6 \text{ and } -5$]

(ii) $x^2 - 9 = (x + 3)(x - 3)$ [Difference of Squares]

Hence simplify

(iii) $\frac{2x^2+x-15}{x^2-9} = \frac{(x+3)(2x-5)}{(x+3)(x-3)} = \frac{2x-5}{x-3}$

◯ Exercise 3.

Express the following as single fractions:

(i) $\frac{7}{6x} + \frac{3}{x} = \frac{7}{6x} + \frac{18}{6x} = \frac{25}{6x}$ [same denominator $= 6x$]

(ii) $\frac{7}{6x^2} - \frac{3}{x^2} = \frac{7}{6x^2} - \frac{18}{6x^2} = -\frac{11}{6x^2}$ [same denominator $= 6x^2$]

Hence simplify

(iii) $\left(\frac{7}{6x} + \frac{3}{x}\right) \div \left(\frac{7}{6x^2} - \frac{3}{x^2}\right) = \frac{25}{6x} \div -\frac{11}{6x^2} = \frac{25}{6x} \times -\frac{6x^2}{11} = -\frac{25x}{11}$